IEEE 18th International Symposium on Applied Computational Intelligence and Informatics
May 21-25, 2024
Siófok, Hungary
Timisoara, Romania

Experiments with a One Parameter Computed Torque Position Control Method Using a PLC and a Simple Linear Actuator

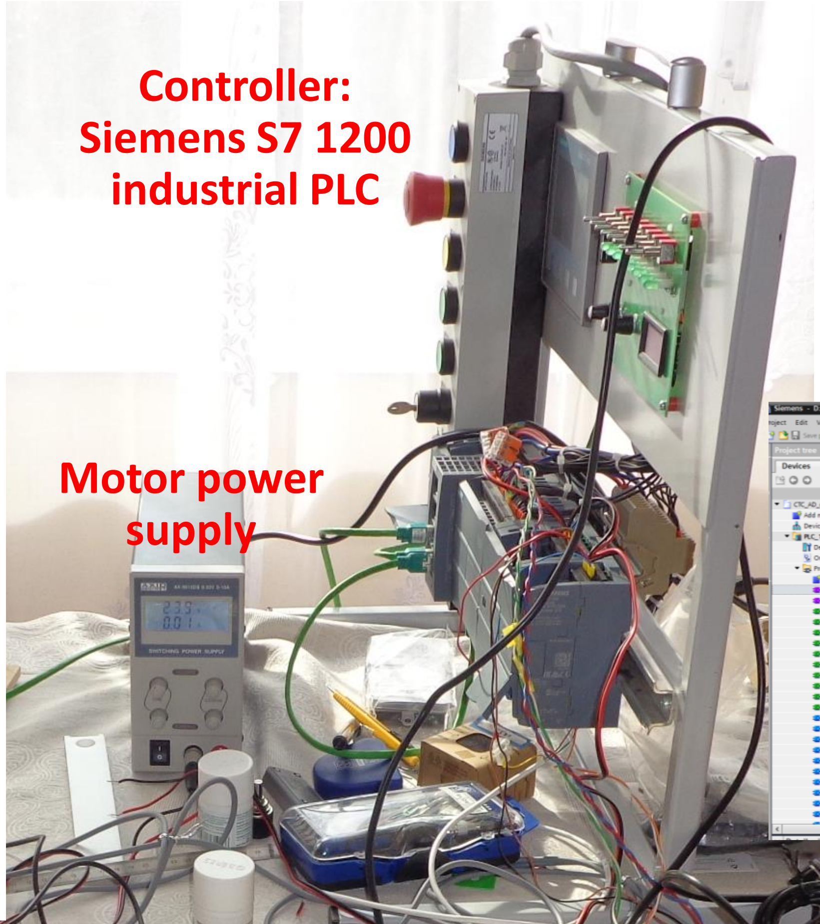
Árpád Varga
Obuda University, Budapest, Hungary
Doctoral School of Applied Informatics and Applied Mathematics
varga.arpad@uni-obuda.hu

János Fricz
Obuda University, Budapest, Hungary
Kando Kalman Faculty of Electrical Engineering
B9QOCM@stud.uni-obuda.hu

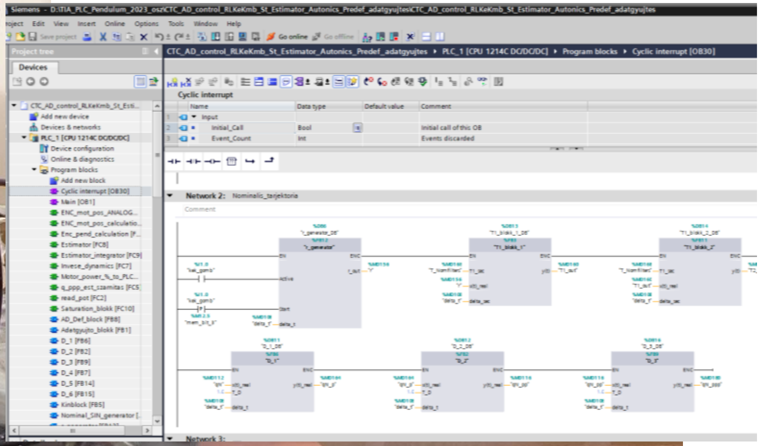
Zoltán Braun
Obuda University, Budapest, Hungary
Kando Kalman Faculty of Electrical Engineering
i7e8ri@stud.uni-obuda.hu

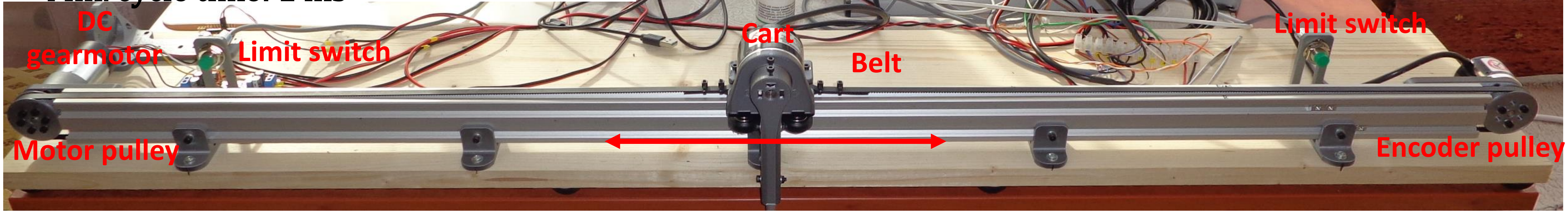
Linear positioning testbed

- Cart moved by a brushed DC motor along a linear rail
- Motor driver is connected to PLC analog output
- Can be converted to pendulum on a cart system
- As many robust industrial components, as possible
- Pos. Resolution: 0.13 mm
- Clogging estimated as ±0.2-0.3 mm
- Min. cycle time: 1 ms

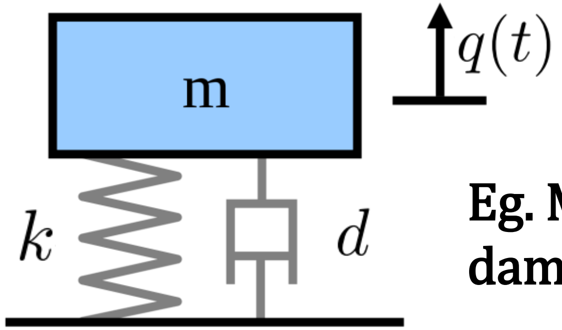


- Software is written in TIA portal environment, control algorithm is inside a cyclic interrupt task





Open-loop CTC



Eg. Mass-spring-damper system

$$q^N(t) = Asin(\omega t)$$

$$\ddot{q}^N(t) = -A\omega^2 sin(\omega t)$$

$$m\ddot{q}(t) + d\dot{q}(t) + kq(t) = Q(t)$$

$$Q^{Des}(t) = m\ddot{q}^N(t) + d\dot{q}(t) + kq(t)$$

From measurement

$$m\ddot{q}(t) + d\dot{q}(t) + kq(t) = m\ddot{q}^N(t) + d\dot{q}(t) + kq(t)$$

$$\ddot{q}(t) = \ddot{q}^N(t)$$

Not practical in real-life applications!

Closed-loop CTC (traditional method)

$$e(t) = q^N(t) - q(t) \qquad \dot{e}(t) = \dot{q}^N(t) - \dot{q}(t)$$

$$\ddot{q}^{Des}(t) = \ddot{q}^N(t) + \underbrace{K_D}_{\text{Derivative gain}}\dot{e}(t) + \underbrace{K_P}_{\text{Proportional gain}}e(t)$$

$$Q^{Des}(t) = m\ddot{q}^{Des}(t) + d\dot{q}(t) + kq(t)$$

$$\cancel{m}\ddot{q}(t) + \cancel{d\dot{q}(t)} + \cancel{kq(t)} = \cancel{m}\ddot{q}^{Des}(t) + \cancel{d\dot{q}(t)} + \cancel{kq(t)}$$

$$\ddot{q}(t) = \ddot{q}^{Des}(t)$$

Closed-loop CTC (traditional method, stability)

$$\ddot{q}(t) = \ddot{q}^N(t) + K_D\dot{e}(t) + K_P e(t)$$

$$\ddot{q}^N(t) - \ddot{q}(t) + K_D\dot{e}(t) + K_P e(t) = 0 \implies$$

$$\ddot{e}(t) + K_D\dot{e}(t) + K_P e(t) = 0.$$

Characteristic equation:

$$\lambda^2 + K_D\lambda + K_P = 0$$

$$\lambda_{1,2} = -\frac{1}{2}K_D \pm \frac{1}{2}\sqrt{K_D^2 - 4K_P}$$

$$e(t) = C_1 e^{\lambda_1(t)} + C_2 e^{\lambda_2(t)}$$

Stable for all positive K_D, K_P values!

- no integral term
- two parameter to tune

Closed-loop CTC (alternative method)

Operator

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)$$

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right) e(t) := \Lambda e(t) + \dot{e}(t)$$

$$e_{int}(t) = \int_0^t e(t)\, dt$$

$$\left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^3 e_{int}(t) =$$

$$\left(\Lambda^3 + 3\Lambda^2 \frac{\mathrm{d}}{\mathrm{d}t} + 3\Lambda \frac{\mathrm{d}^2}{\mathrm{d}t^2} + \frac{\mathrm{d}^3}{\mathrm{d}t^3}\right) e_{int}(t) =$$

$$\Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{e}(t).$$

$$\ddot{q}^N$$

$$\ddot{q}^{Des}(t) = \Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{q}^N(t)$$

Closed-loop CTC (alternative method)

$$\ddot{q}^{Des}(t) = \Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{q}^N(t) \qquad Q^{Des}(t) = m\ddot{q}^{Des}(t) + d\dot{q}(t) + kq(t)$$

$$m\ddot{q}(t) + d\dot{q}(t) + kq(t) =$$

$$m\left[\Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{q}^N(t)\right] + \ldots$$

$$\ldots + d\dot{q}(t) + kq(t) \implies$$

$$\Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{e}(t) = 0$$

$$\ddot{e}(t) = -\Lambda^3 e_{int}(t) - 3\Lambda^2 e(t) - 3\Lambda \dot{e}(t) \implies \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$$

$$\begin{bmatrix} \ddot{e}(t) \\ \dot{e}(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} -3\Lambda & -3\Lambda^2 & -\Lambda^3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{e}(t) \\ e(t) \\ e_{int}(t) \end{bmatrix}$$

State spece representation

Closed-loop CTC (alternative method, stability)

Real parts of all eigenvalues must be positive

$$\det(\mathbf{A} - \lambda\mathbf{I}) =$$

$$\det\left(\begin{bmatrix} -3\Lambda - \lambda & -3\Lambda^2 & -\Lambda^3 \\ 1 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{bmatrix}\right) =$$

$$-3\Lambda\lambda^2 - \lambda^3 - 3\Lambda^2\lambda - \Lambda^3 =$$

$$-(\lambda + \Lambda)^3.$$

$$-(\lambda+\Lambda)^3 = 0 \qquad \lambda_{1,2,3} = -\Lambda \qquad \lim_{t\to\infty} \mathbf{x}(t) = \mathbf{0}$$

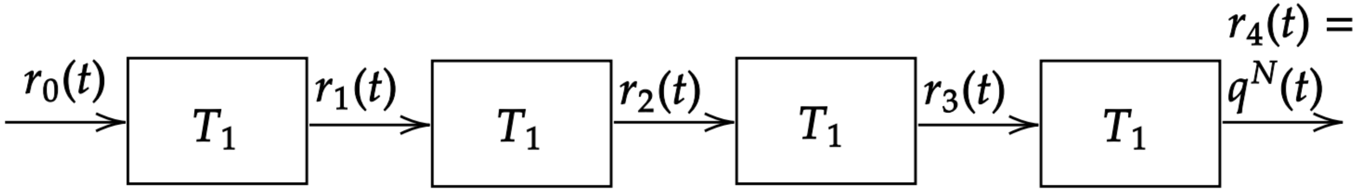
Stable for all positive Λ values!

Nominal trajectory generation (if there is no pre-defined one)

- Simple logical condition for the reference signal:

$$r_0(t) = \begin{cases} 0.15 \text{ if } t < 1.3 \\ 0.0 \text{ otherwise,} \end{cases}$$

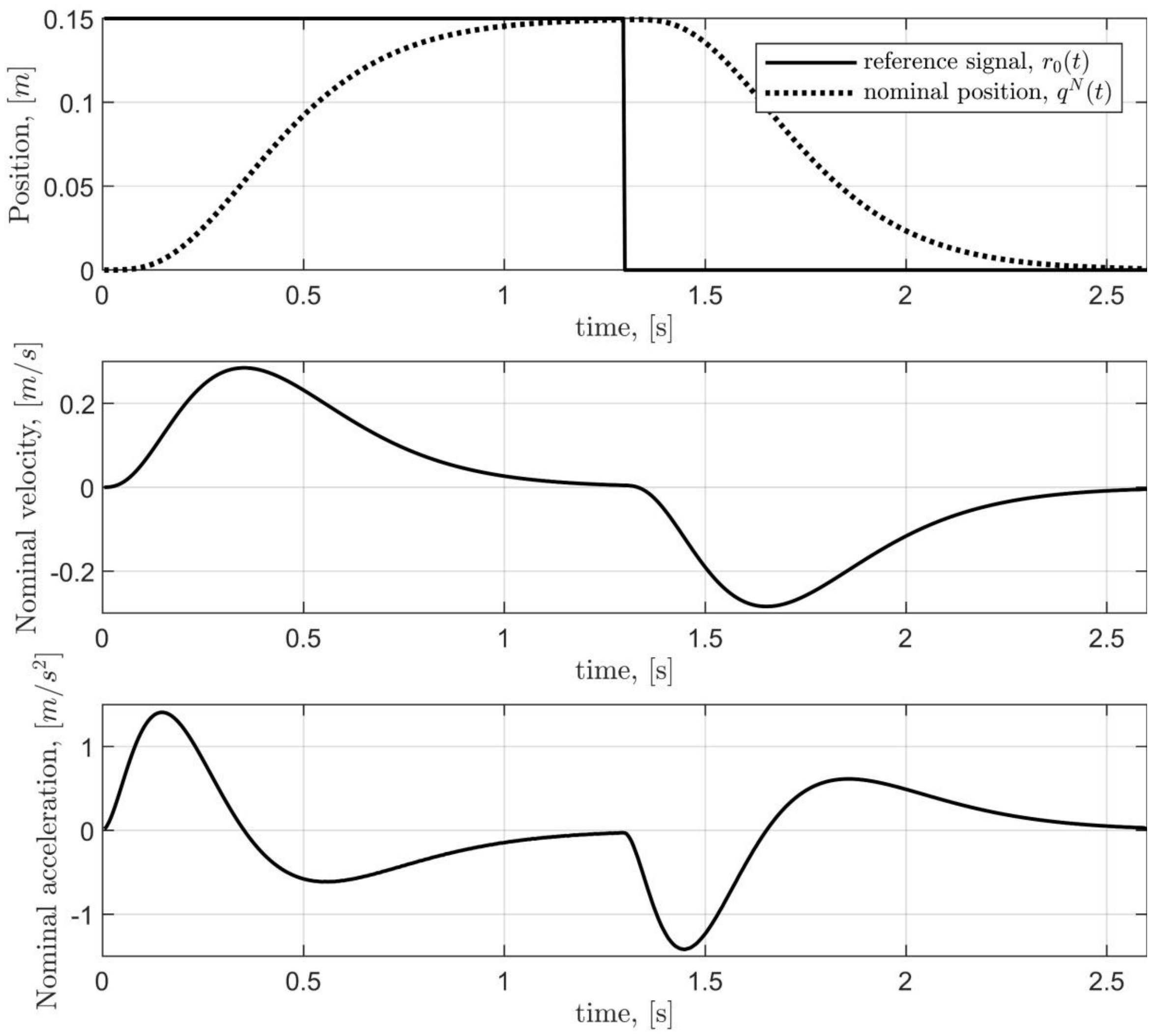
- Reference „pre-filtering” to get smooth nominal tarjectory:



- State-space reprersentation of the trajectory generation process:

$$\begin{bmatrix} \dot{r}_1(t) \\ \dot{r}_2(t) \\ \dot{r}_3(t) \\ \dot{r}_4(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau} & 0 & 0 & 0 \\ \frac{1}{\tau} & -\frac{1}{\tau} & 0 & 0 \\ 0 & \frac{1}{\tau} & -\frac{1}{\tau} & 4 \\ 0 & 0 & \frac{1}{\tau} & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} r_1(t) \\ r_2(t) \\ r_3(t) \\ r_4(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{\tau} \\ 0 \\ 0 \\ 0 \end{bmatrix} r_0(t).$$

- Smooth velocity and accelaration curves:



System identification

- „mbK" model:

$$m_{red}\ddot{q}(t) + b_{red}\dot{q}(t) = Ku(t)$$

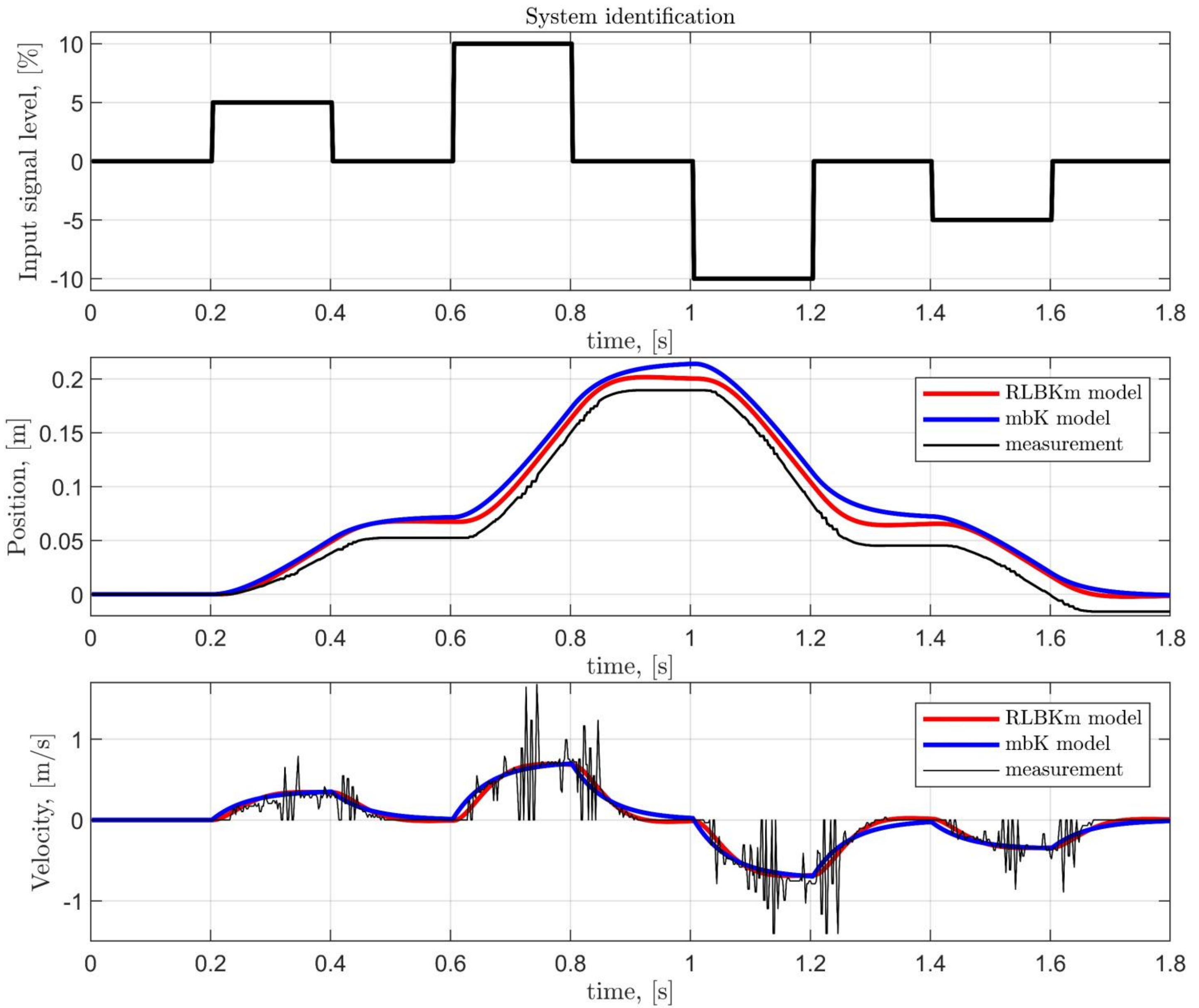
$$\begin{bmatrix} \ddot{q}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} \frac{b_{red}}{m_{red}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \frac{K}{m_{red}} \\ 0 \end{bmatrix} u(t)$$

- „RLBK" model:

$$L\dot{I}(t) + RI(t) + K_e\dot{q}(t) = u(t)$$

$$m_{red}\ddot{q}(t) + b_{red}\dot{q}(t) - K_m I(t) = 0$$

$$\begin{bmatrix} \dot{I}(t) \\ \ddot{q}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K_e}{L} & 0 \\ \frac{K_m}{m_{red}} & -\frac{b_{red}}{m_{red}} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I(t) \\ \dot{q}(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} u(t)$$



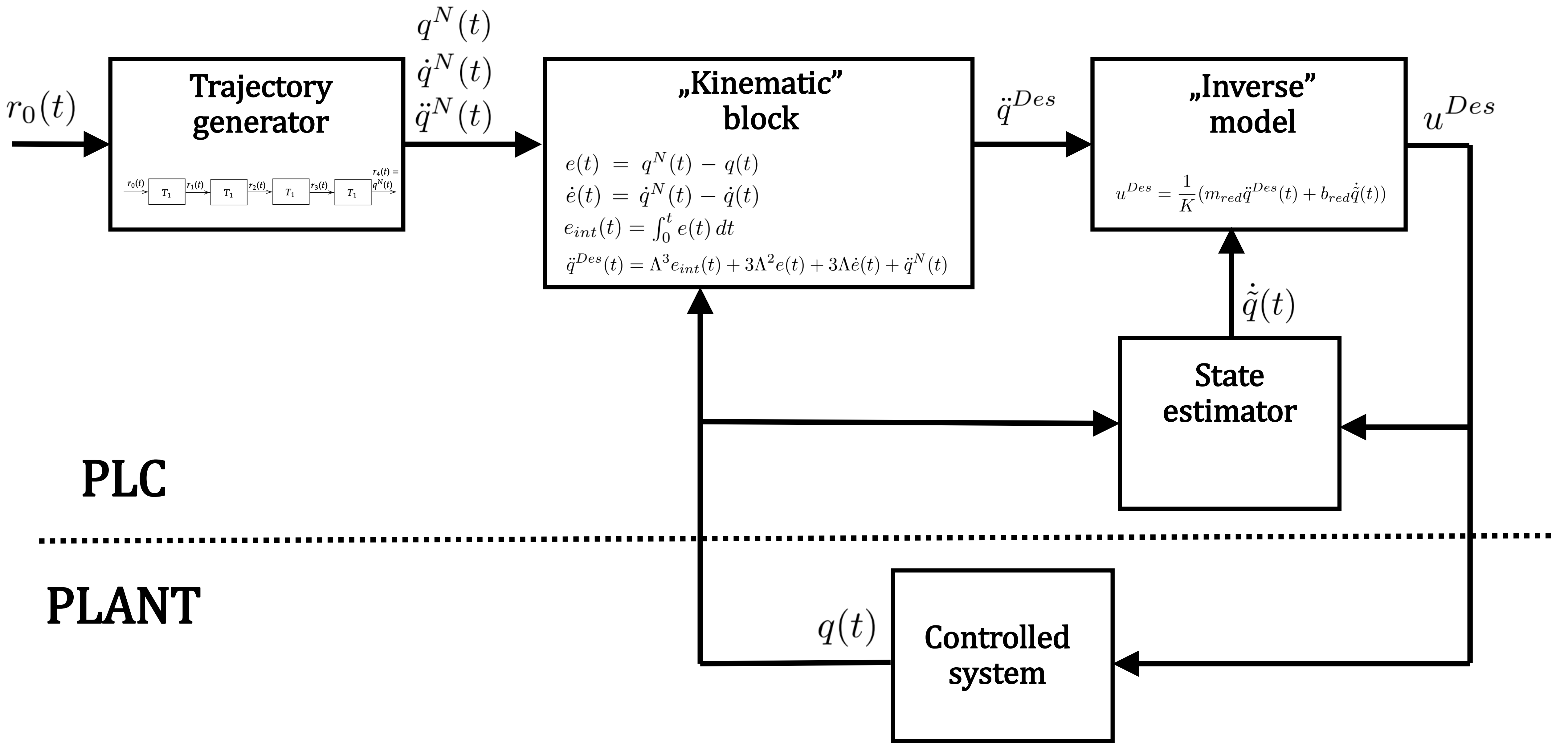
State estimator for filtering out position and velocity signal noise

$$\begin{bmatrix} \ddot{\tilde{q}}(t) \\ \dot{\tilde{q}}(t) \end{bmatrix} = \begin{bmatrix} \frac{b_{red}}{m_{red}} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\tilde{q}}(t) \\ \tilde{q}(t) \end{bmatrix} + \begin{bmatrix} \frac{K}{m_{red}} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (q - \tilde{q})$$

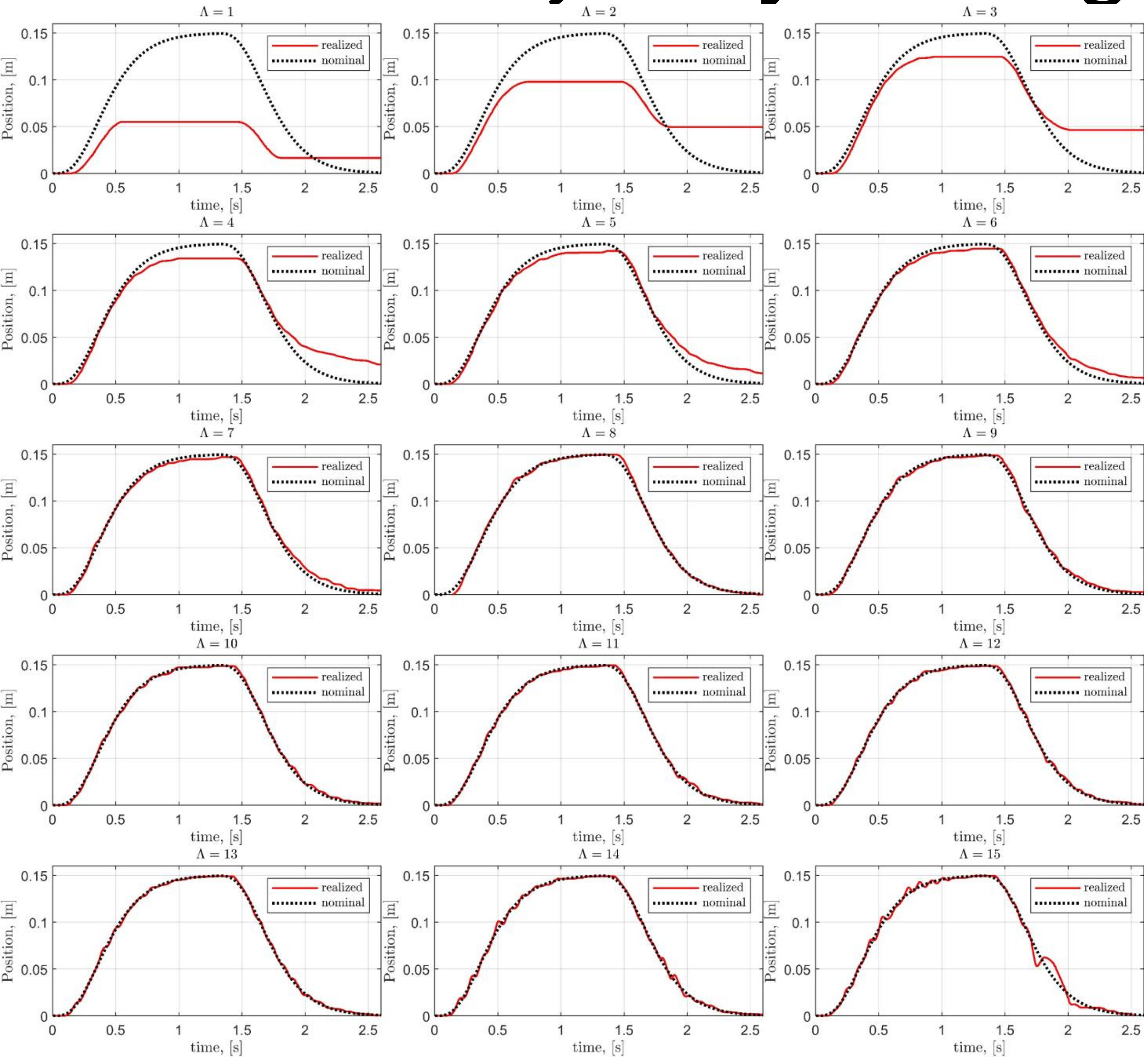
- „mbK” model:

$$u^{Des} = \frac{1}{K}(m_{red}\ddot{q}^{Des}(t) + b_{red}\dot{\tilde{q}}(t))$$

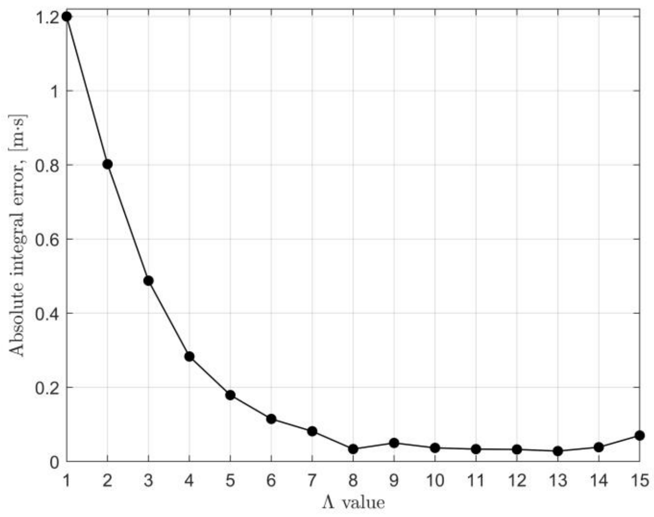
Full realized CTC scheme



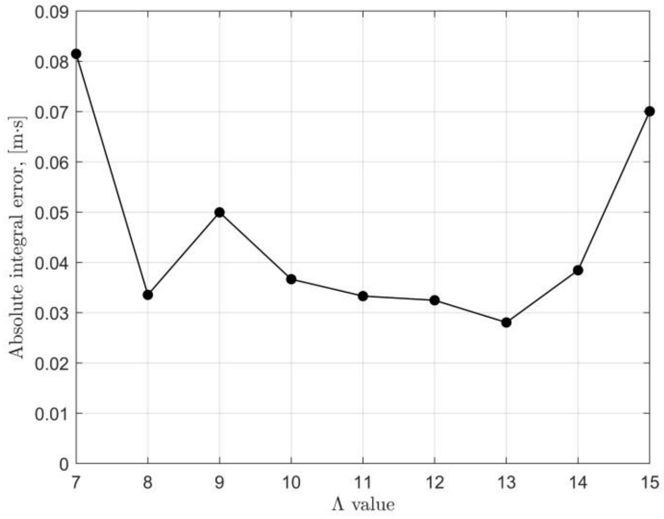
Trajectory tracking for Λ=1…15



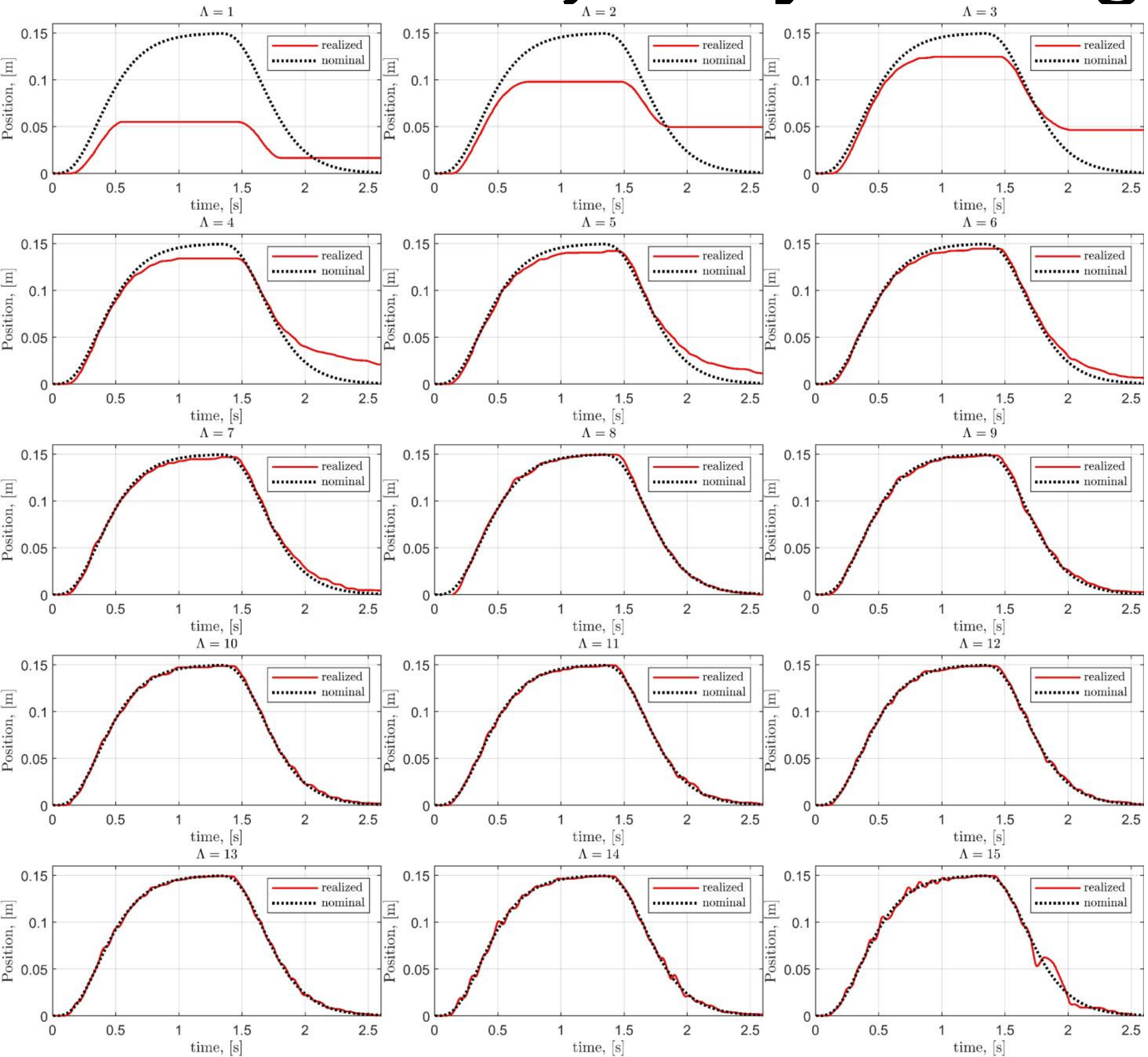
Integral error, Λ=1…15

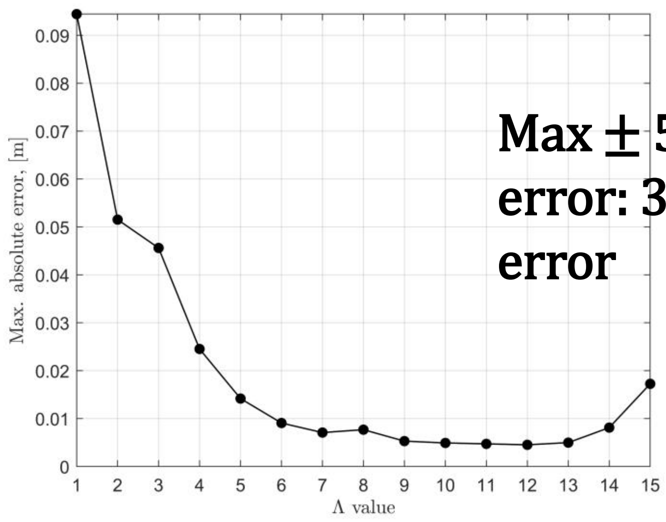


Integral error, Λ=7…15

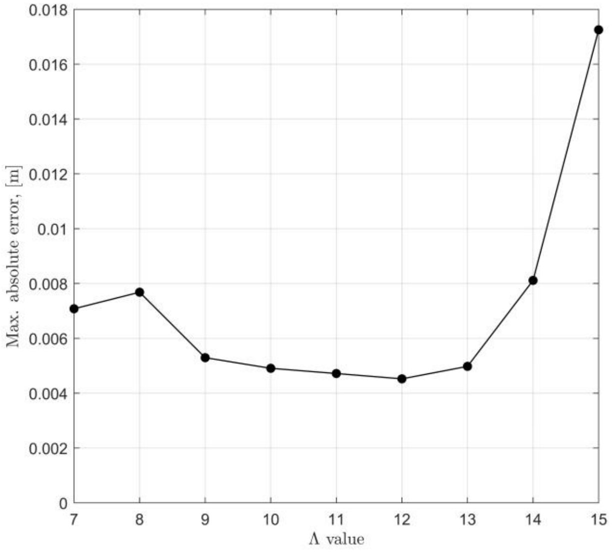


Trajectory tracking for Λ=1..15

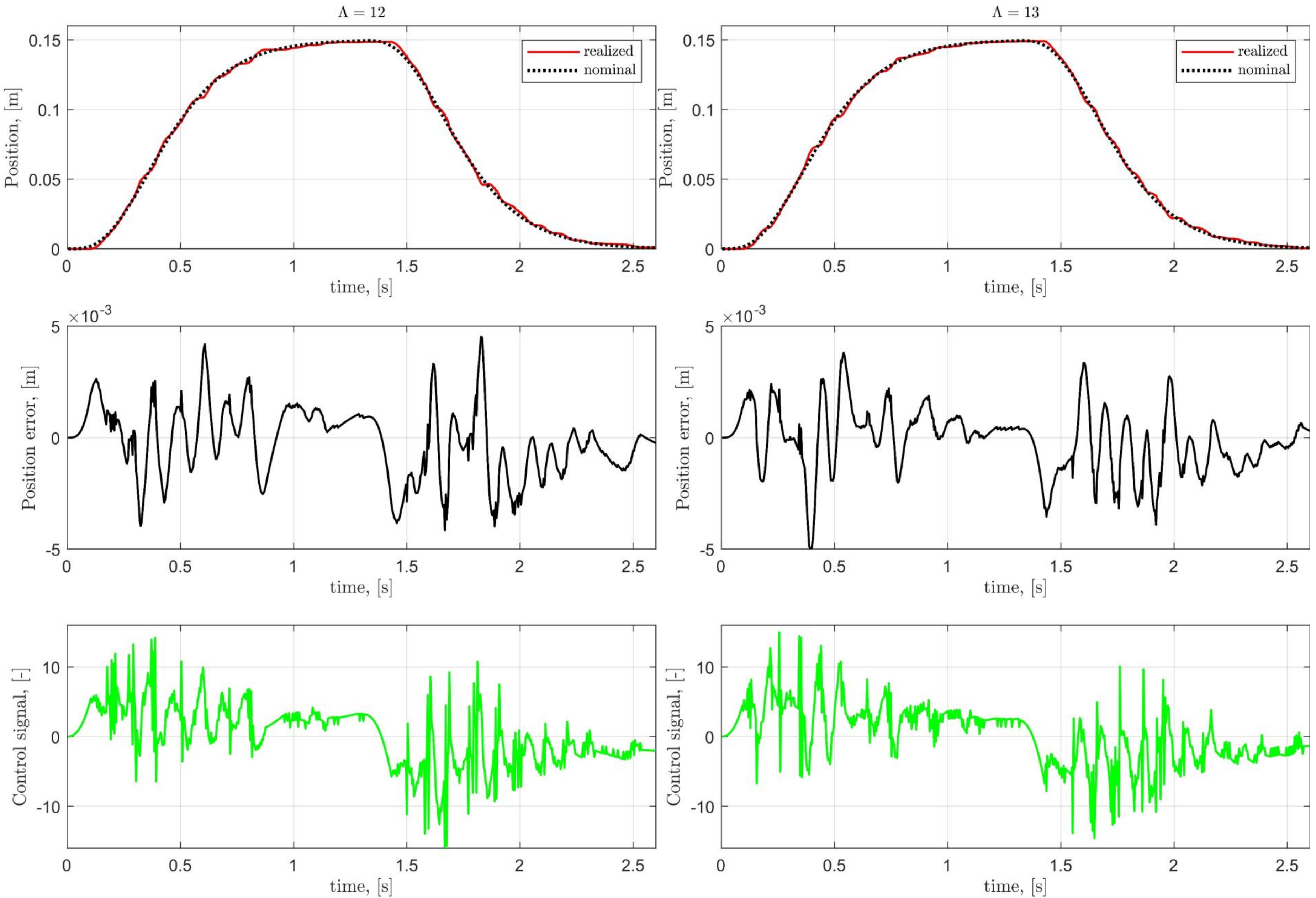




Max ± 5 mm error: 3% relative error

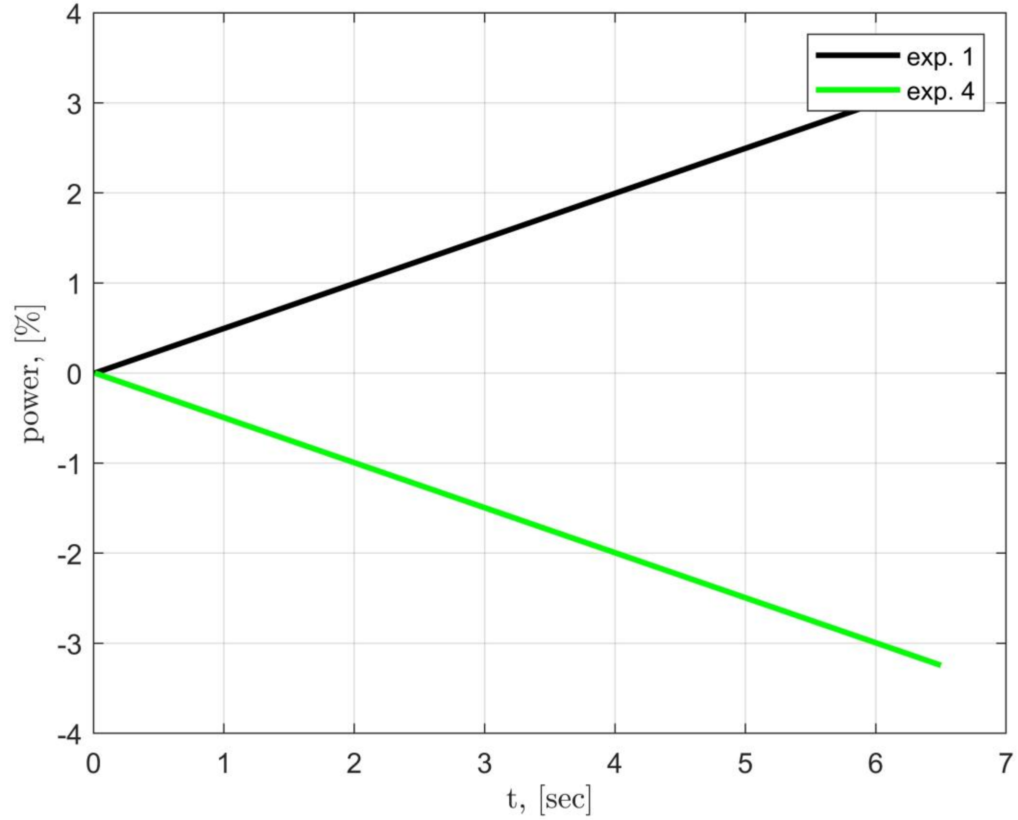


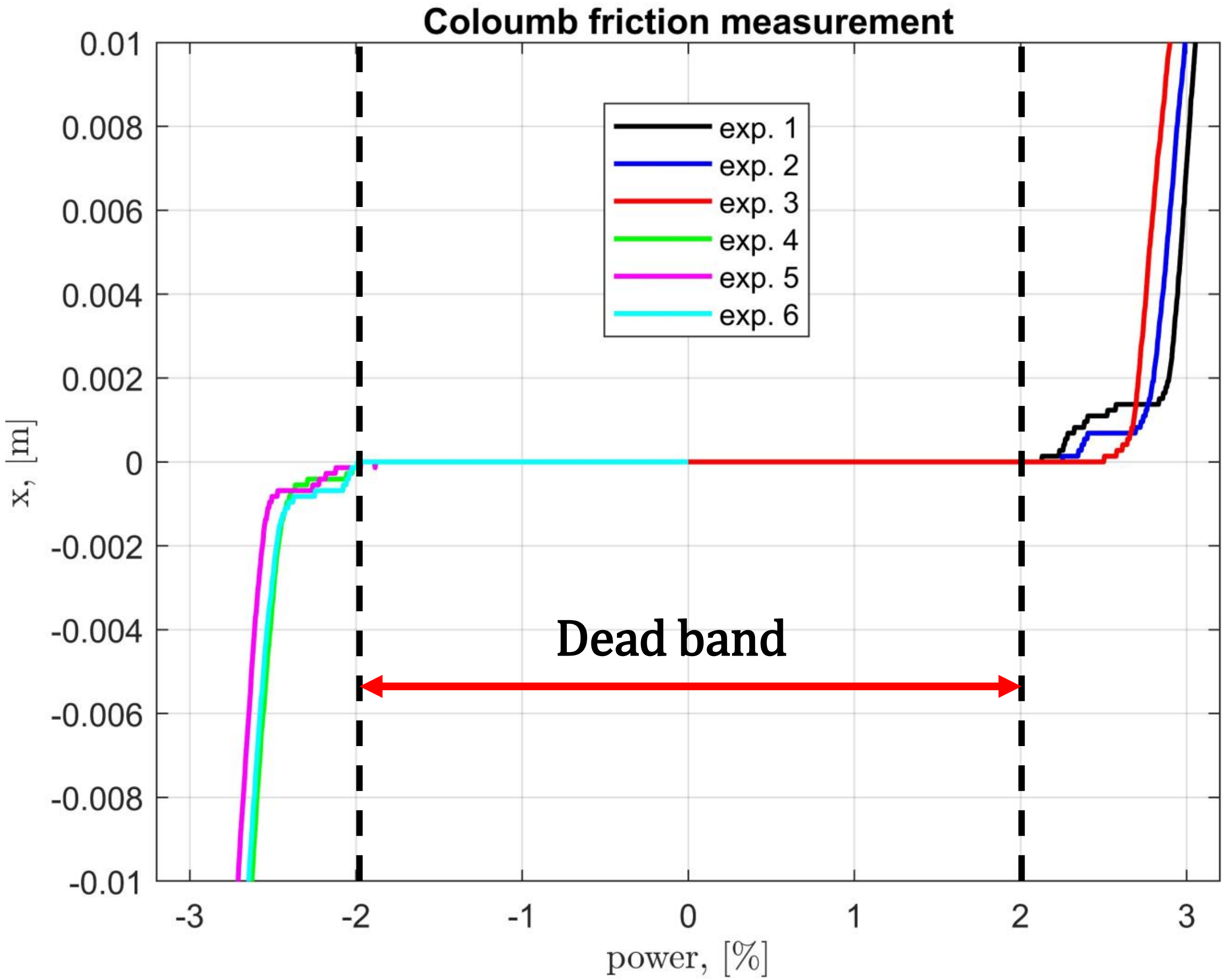
Best trajectory tracking results Λ=12..13



Possible explanations for oscillations at high Λ values:

- Unmodelled Coloumb friction, proven by a measurement:



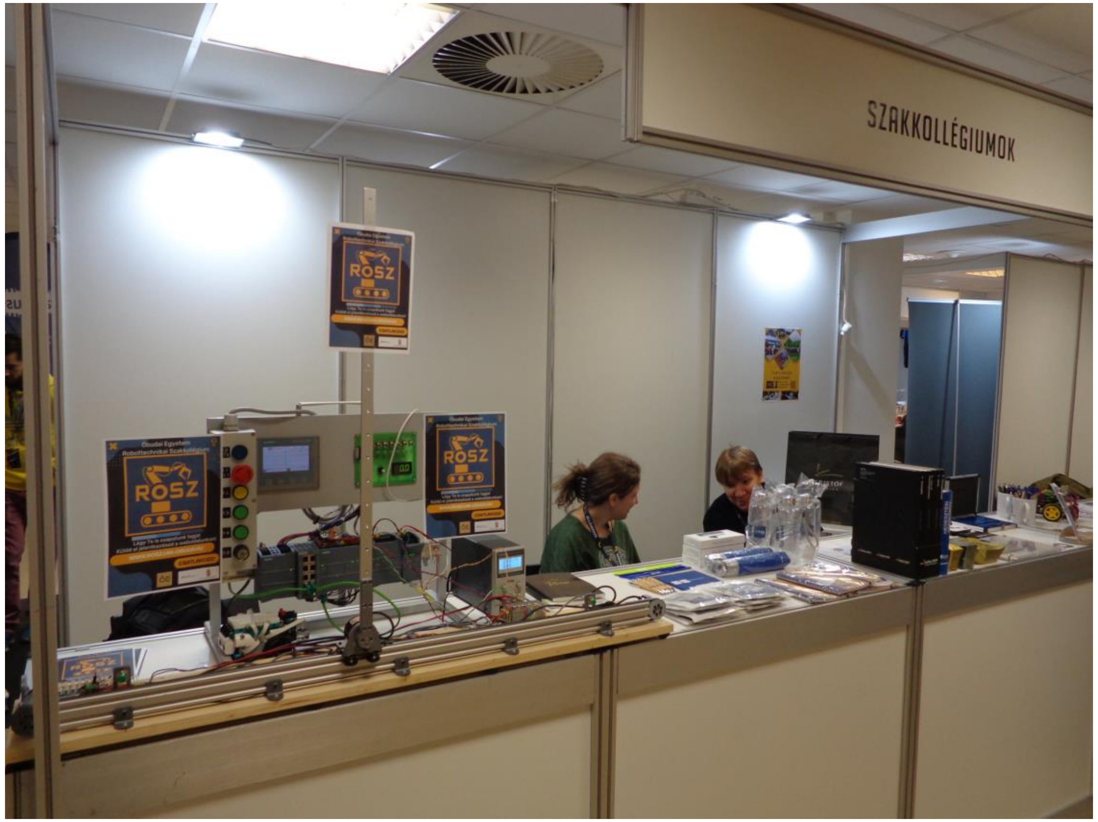


Demonstration at different events:

The publication was prepared in the framework of the grant **NTP-SZKOLL-23-0051 (Szakkollégiumok tehetséggondozó programjainak támogatása)** of the Robotics Students Association of Óbuda University (ROSZ). The experimental device presented can be converted into an inverted pendulum, which is regularly used at events to promote the ROSZ .

Kutatók éjszakája

OE Állásbörze



Thank you for your attention!