Programming a PID controller and a closed-loop control simulation using a PLC

Mini course in Embedded Systems
2023.11.7-28.

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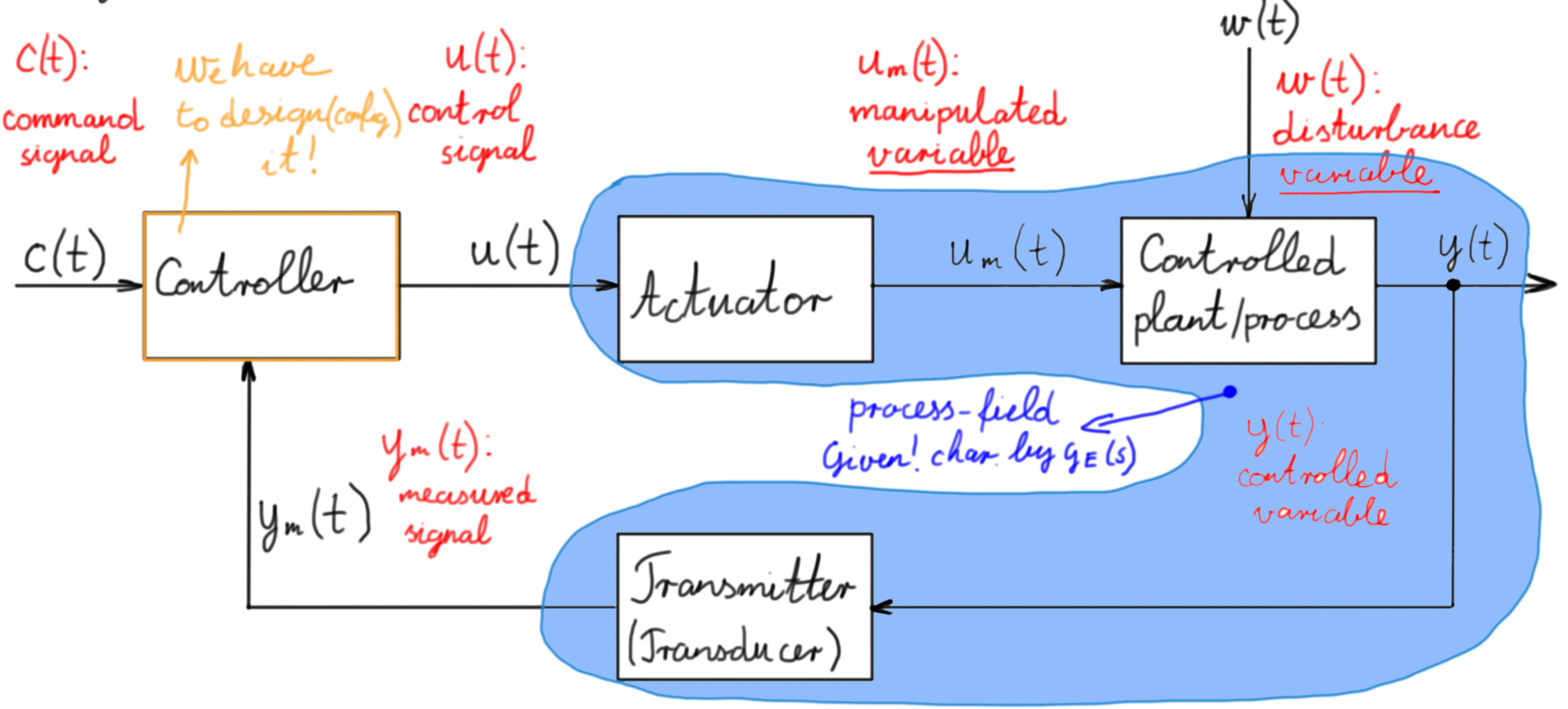
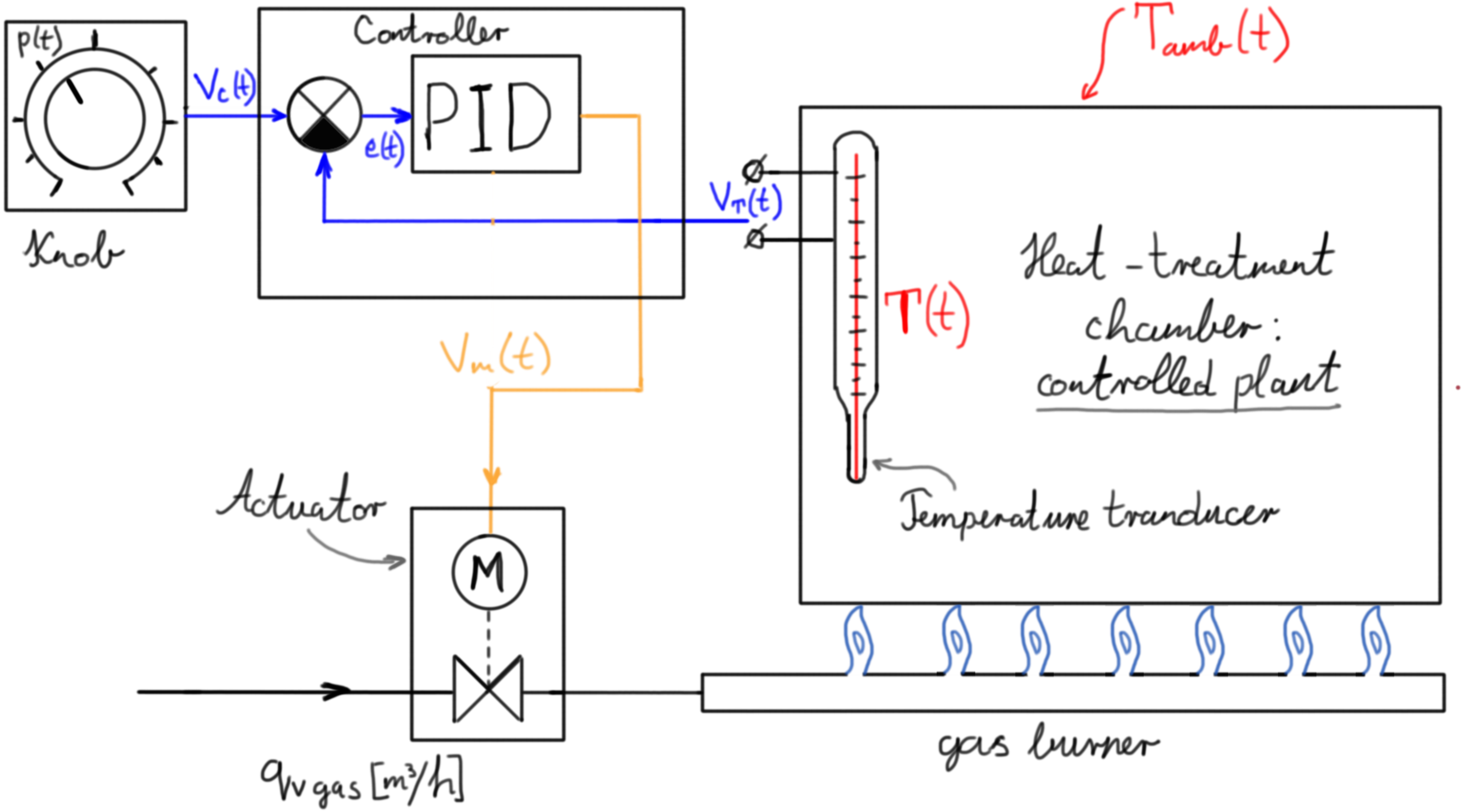
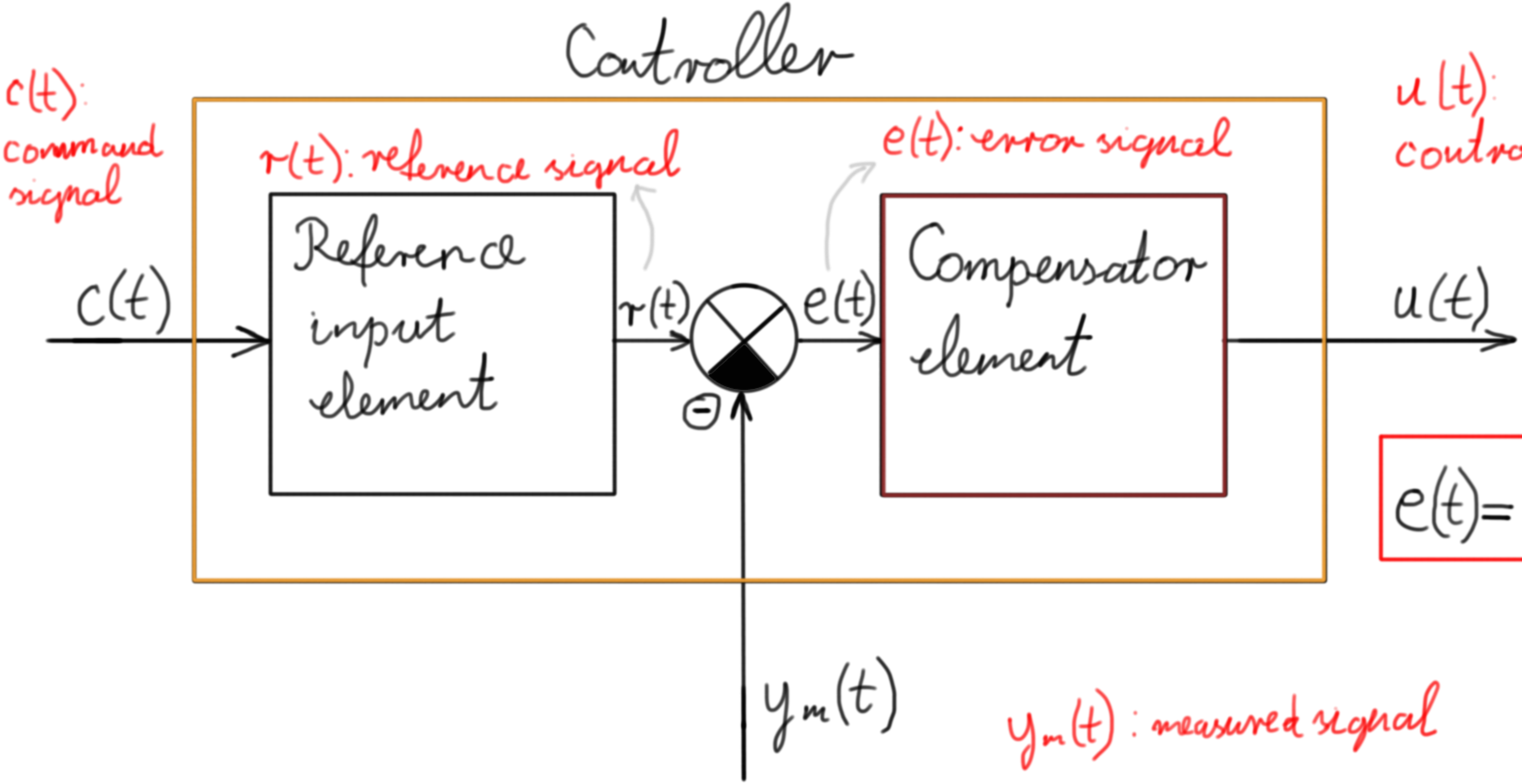
Signals, variables and elements of a closed-loop control





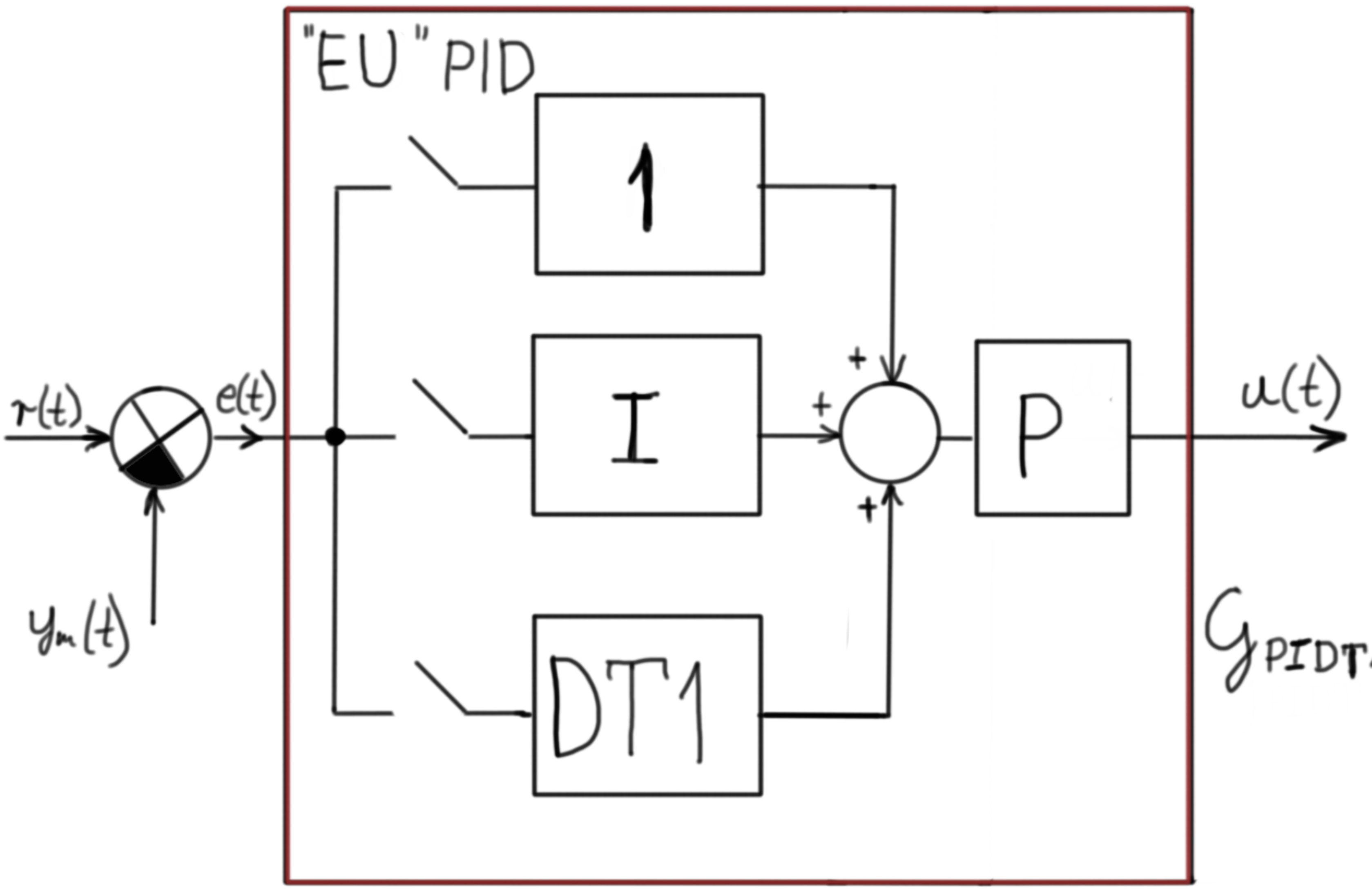
p(t)
Knob
Controller
PID
$V_c(t)$
e(t)
$V_T(t)$
$V_m(t)$
$T_{amb}(t)$
T(t)
Heat-treatment chamber: controlled plant
Temperature tranducer
Actuator
M
q_v gas [m³/h]
gas burner

Inside the controller:



$$\boxed{e(t) = r(t) - y_m(t)}$$

Compensator element: 3 switchable (P, I, D) channels, EU/US structure:

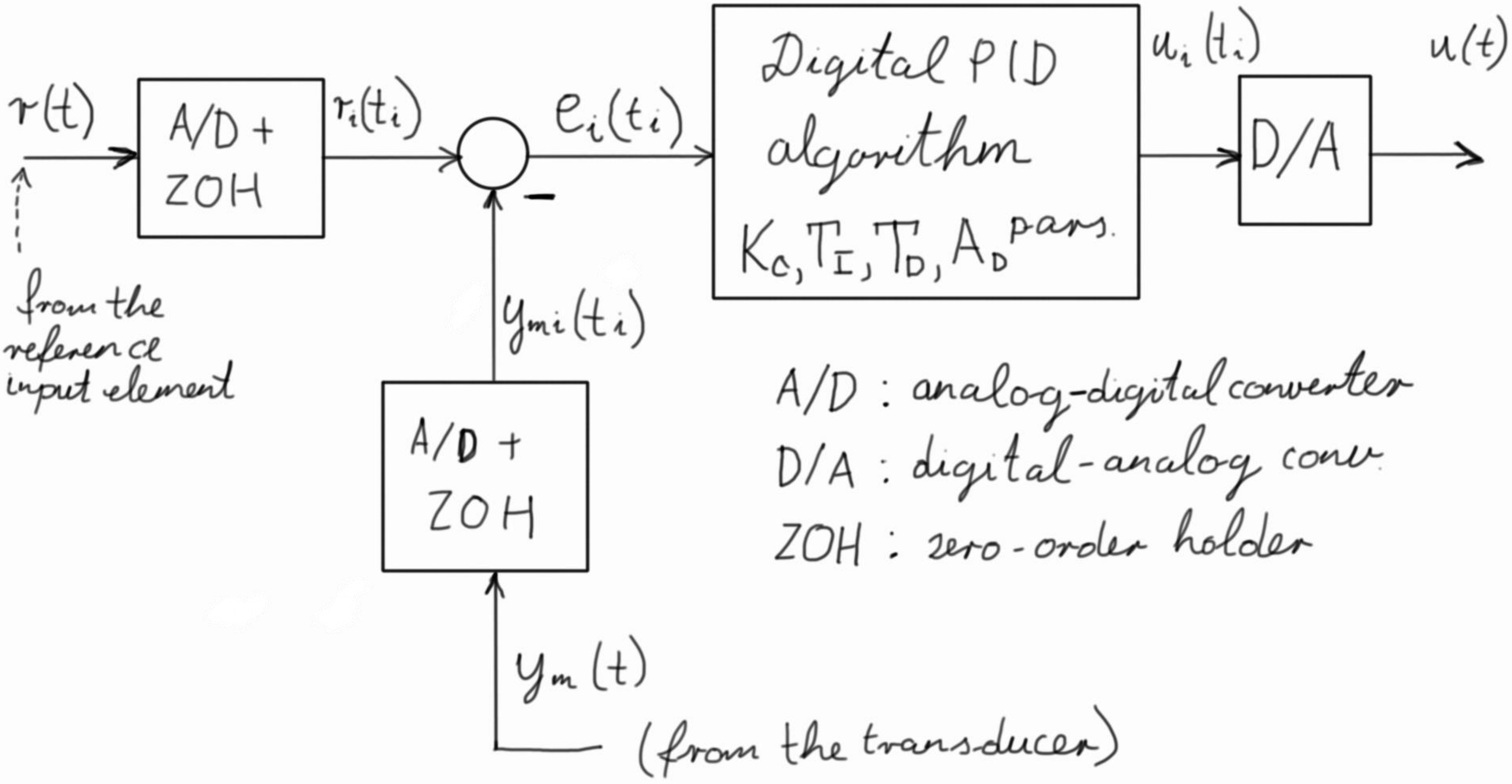


$$G_P(s) = K_C$$

$$G_I(s) = \frac{1}{sT_I}$$

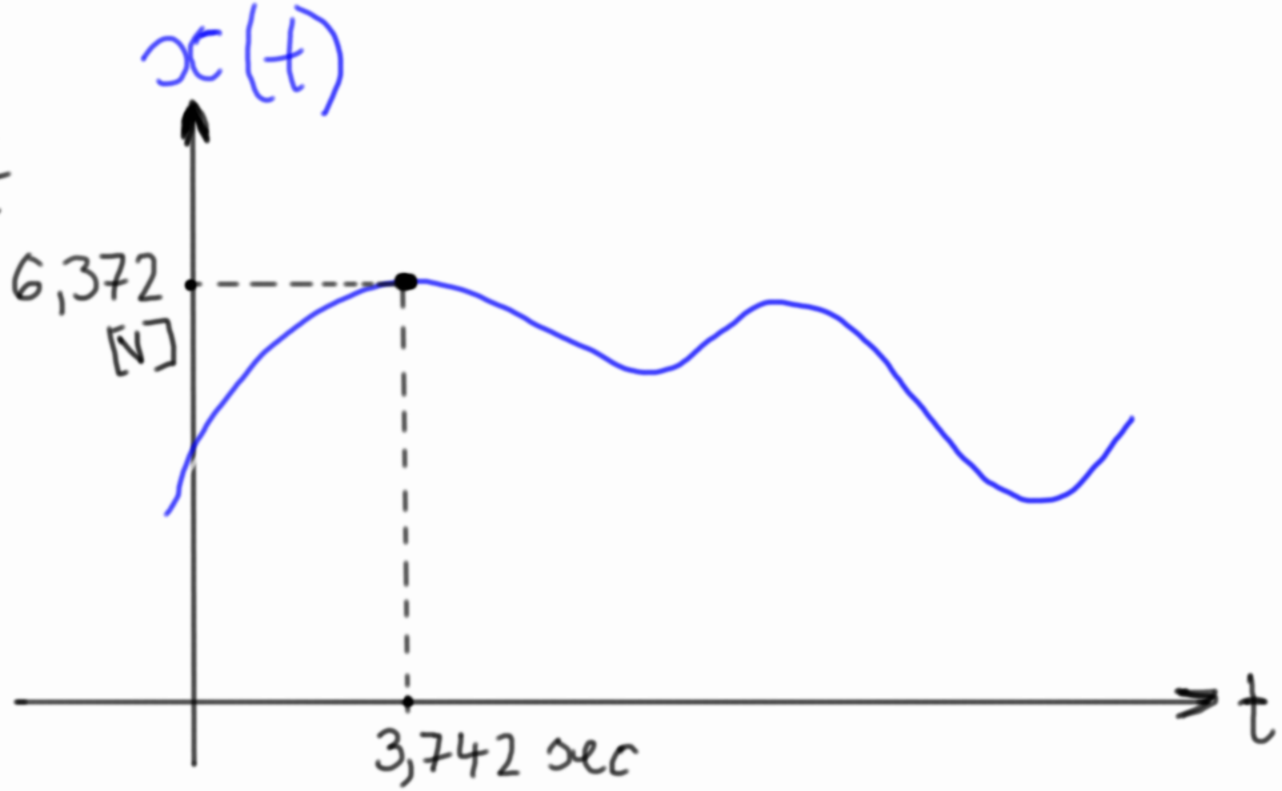
$$G_{DT1}(s) = \frac{sT_D}{sT_1 + 1}$$

$$G_{PIDT1}(s) = \left[1 + G_I(s) + G_{DT1}(s)\right] \cdot K_C$$


r(t)
A/D + ZOH
from the reference input element
$r_i(t_i)$
–
$e_i(t_i)$
$y_{mi}(t_i)$
A/D + ZOH
$y_m(t)$
(from the transducer)
Digital PID algorithm
K_c, T_I, T_D, A_D pars.
$u_i(t_i)$
D/A
u(t)
A/D : analog-digital converter
D/A : digital-analog conv.
ZOH : zero-order holder

Continous analog signal:
- interpreted at any t time instant
- x can be any real number

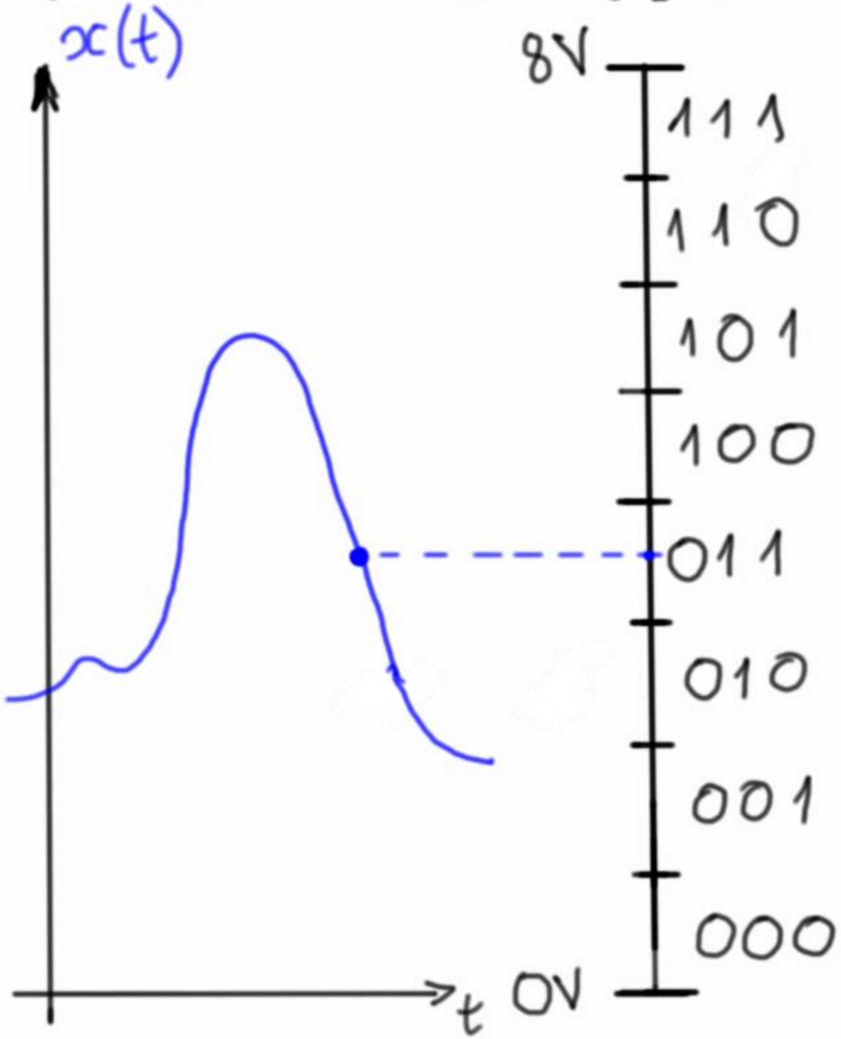
Digital (discrete) signal:
- interpreted only at specific t_i time intances
- x can take values from specific set of values.



Quantification in $\underbrace{\text{value}}_{\text{A/D}}$ and $\underbrace{\text{time}}_{\text{ZOH}}$!

Quantifying in value. Eg. using a 3 bit A/D converter

· Some tranducer provides an analog output signal varying between 0 - 8[V]



· dividing the 0-8V range to 8 sub-range, identified by a 3 bit binary number.

· coercing the actual analog signal value to the nearest digital number

eg. $x(t) = 2{,}7 \approx 3 = 011^{(b)}$

· the digitalized value can take only the values, which is defined by the sub-domain identifiers!

When using n bits in A/D we can devide the whole range of the analog signals into 2^n sub-ranges, where n is the bit number of the A/D converter:

n bit $\mapsto 2^n$ sub-domain

e.g. 3 bits $\longmapsto 2^3 = 8$ sub-domain

4 bits $\longmapsto 2^4 = 16$ —— " ——

commonly used A/D converters:

10 bit : $2^{10} = 1024$ —— " ——

12 bit : $2^{12} = 4096$

14 bit ⋯ 16384

16 bit ⋯ 65536

The smallest recognizable signal change is the resolution of the A/D converter.

eg. 0 - 8V, 3 bit A/D : 1 volt

eg. 0 - 8V, 4 bit A/D : $8V/2^4 = 8V/16 = 0{,}5V$

0 - 10V, 10 bit A/D : $10V/2^{10} = 10V/1024 =$

$= 0{,}0098V \approx 0{,}01$

Quantification in time (sampling)

- the signal values <u>are not</u> interpreted at any arbitrary time instances!
- they are evaluated only at t_i time instances, where:

$$t_i = i \cdot \Delta t$$

$$i = 0, 1, 2, 3 \ldots n$$

Δt : sampling time

$x_i(t_i)$:

$$t_0 \longmapsto x_0$$

$$t_1 \longmapsto x_1$$

$$t_3 \longmapsto x_3$$

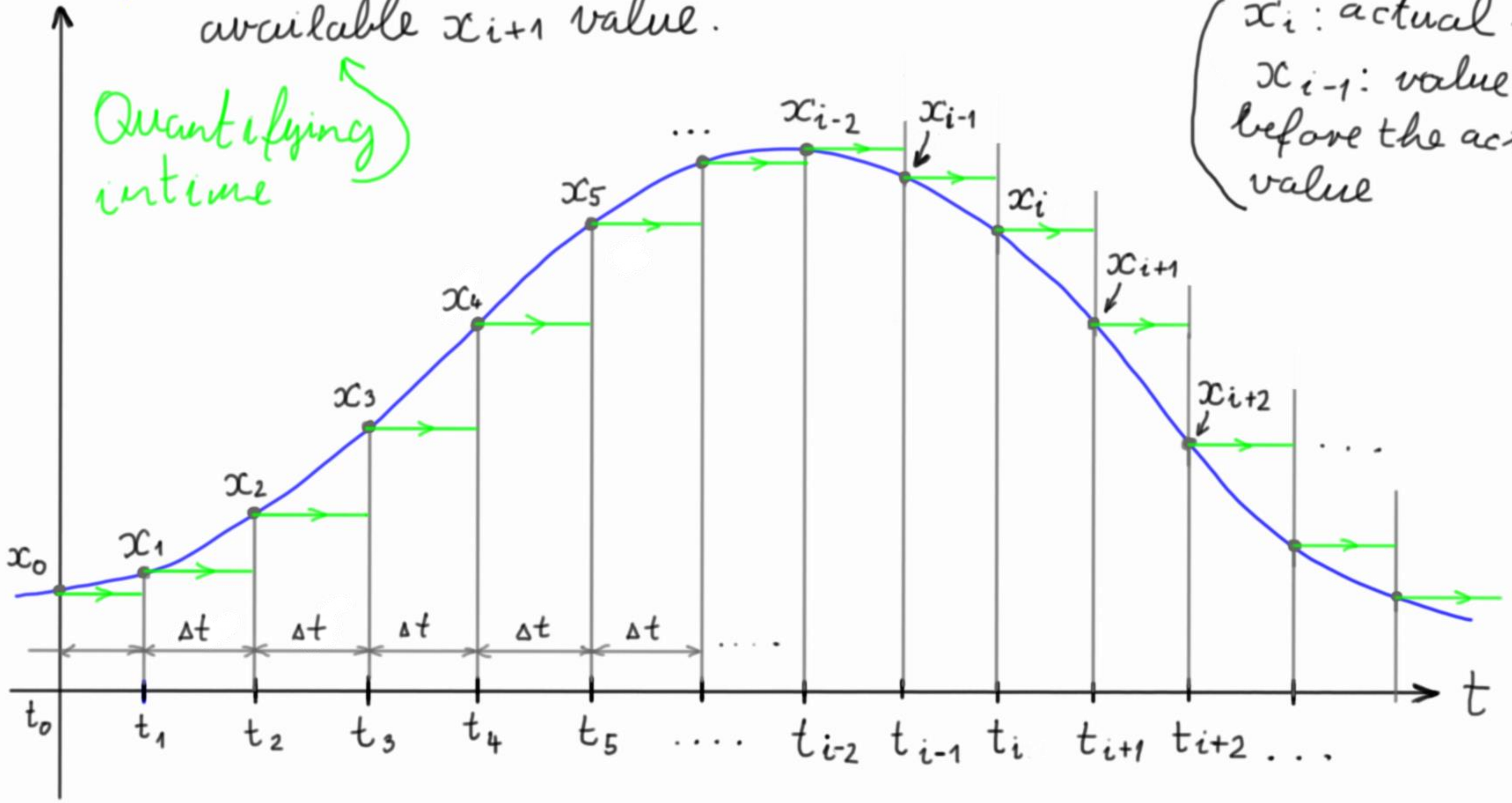
$$\cdots\cdots$$

$$t_n \longmapsto x_n$$

$x(t)$ **ZOH**: the actual x_i kept constant until the next available x_{i+1} value.

(x_i : actual val.
x_{i-1}: value before the actual value)

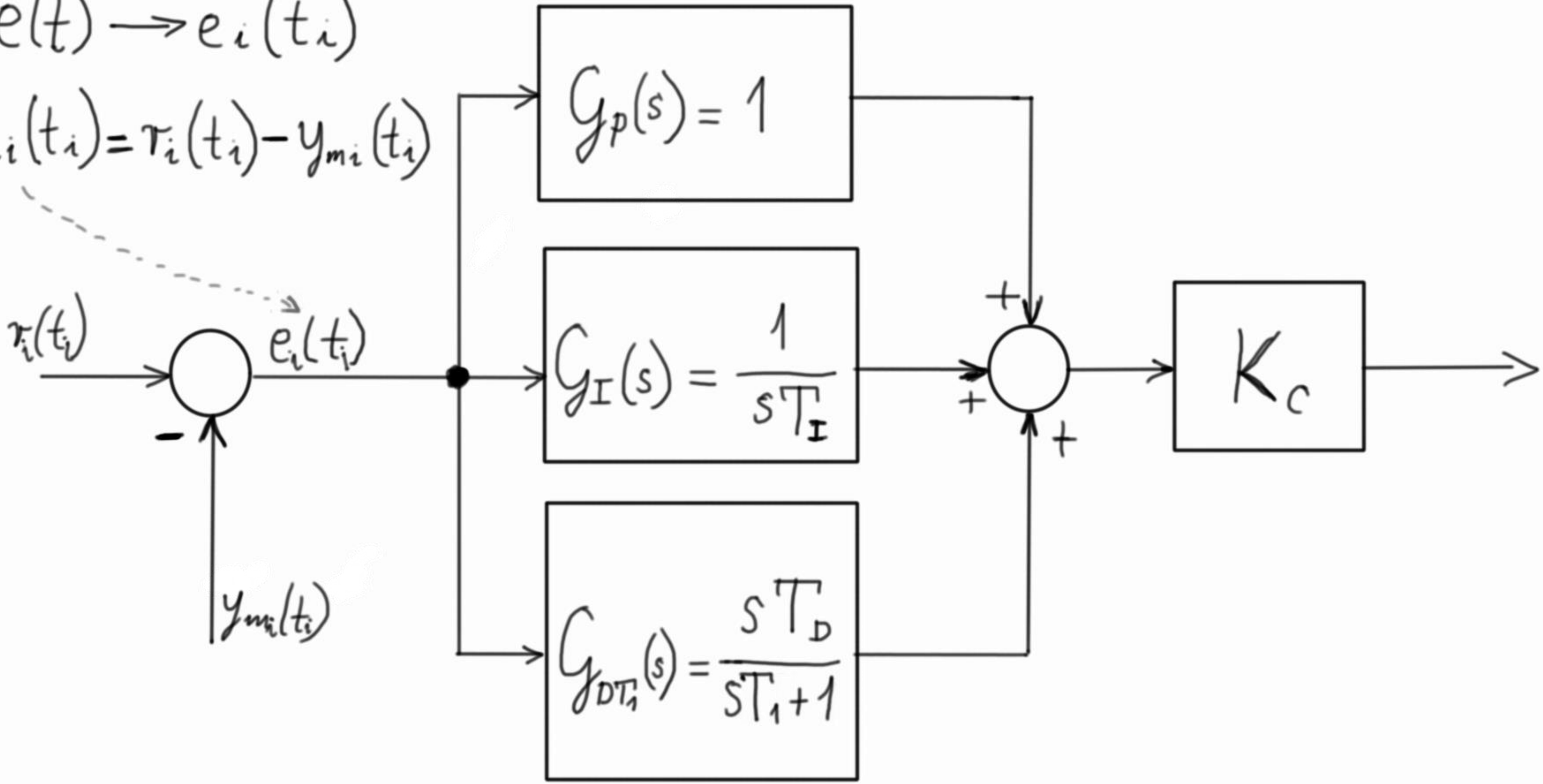
Quantifying in time



Realizing the PID channels (EU structure):

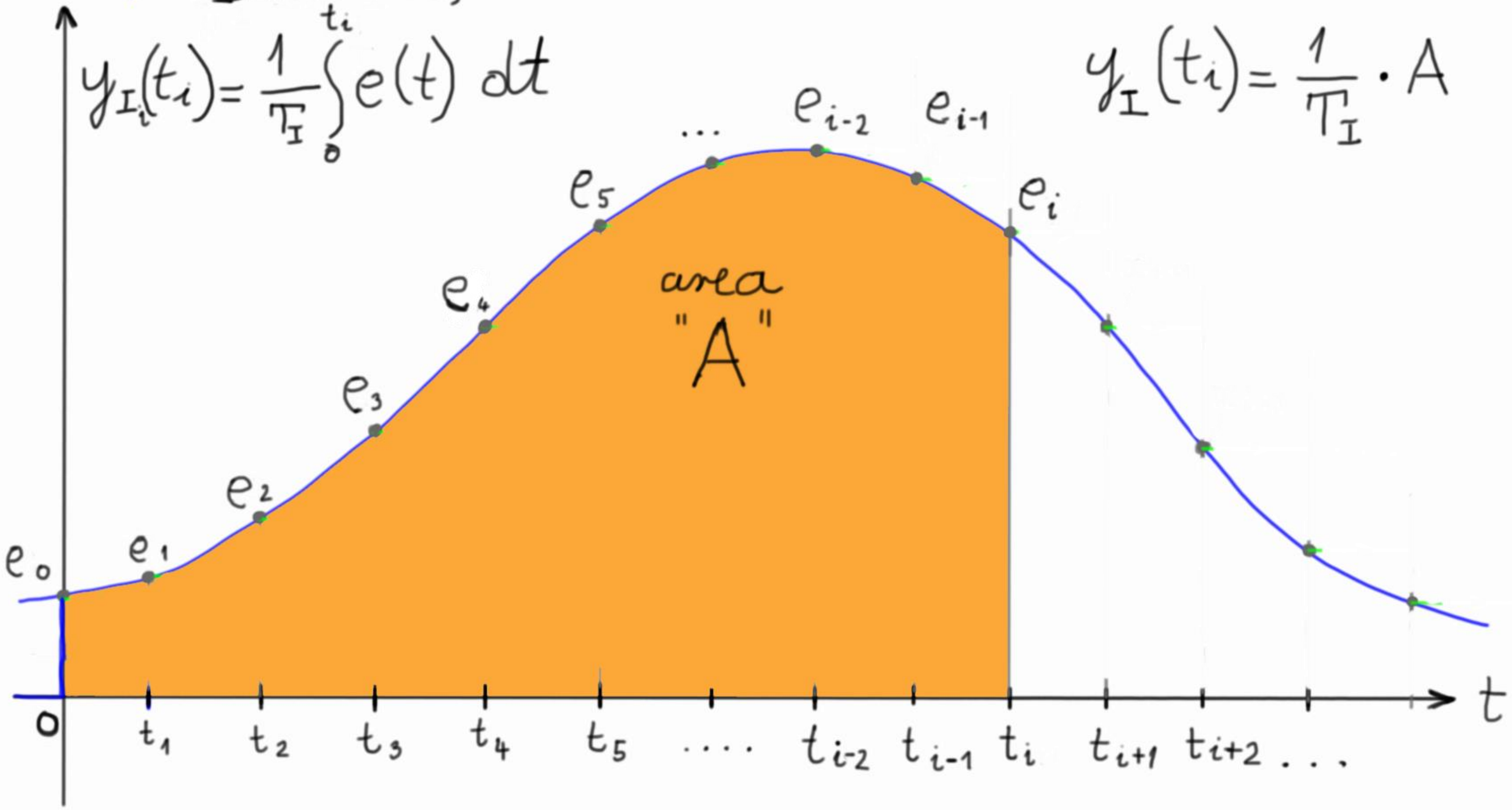
$e(t) \longrightarrow e_i(t_i)$

$e_i(t_i) = r_i(t_i) - y_{mi}(t_i)$

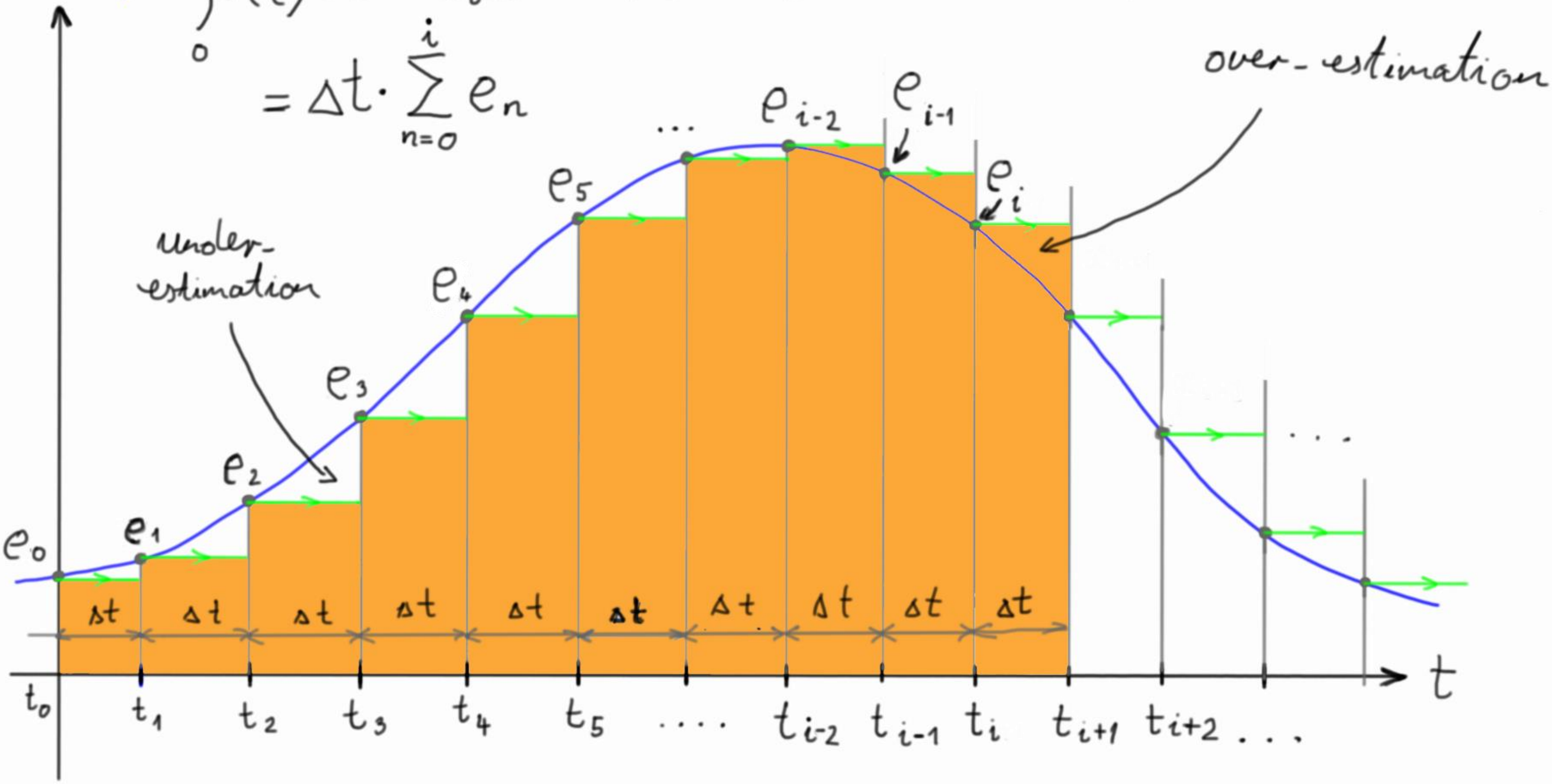


$e(t)$ I channel, continous case:

$$y_I(t_i) = \frac{1}{T_I}\int_0^{t_i} e(t)\,dt \qquad y_I(t_i) = \frac{1}{T_I}\cdot A$$



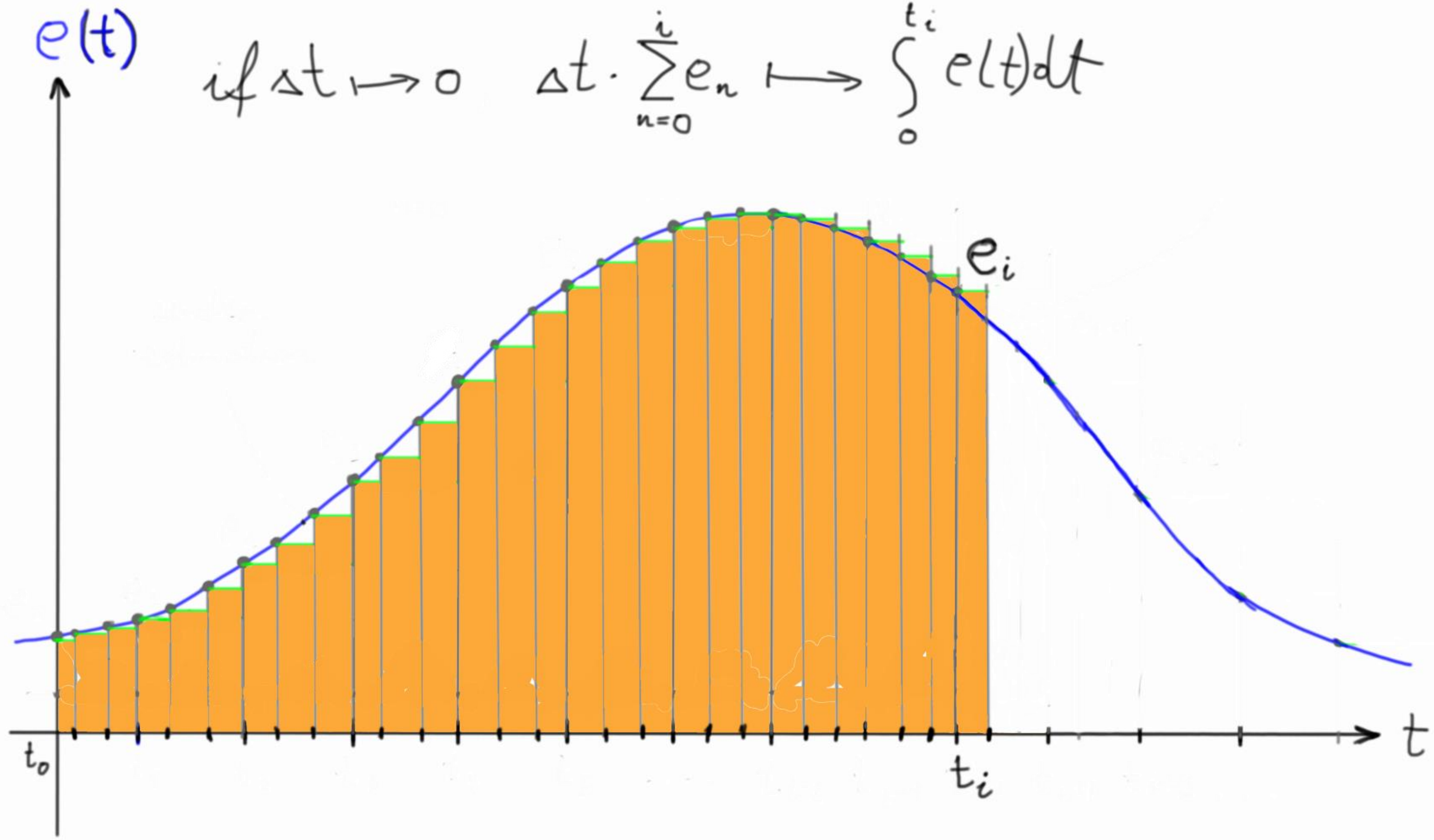
$$\int_0^t e(t)\,dt \approx e_0\Delta t + e_1\Delta t + e_2\Delta t + \ldots + e_{i-1}\Delta t + e_i\cdot\Delta t = \Delta t\cdot\sum_{n=0}^{i} e_n$$



Digital I channel:

$$y_{I_i}(t_i) = \Delta t \cdot \frac{1}{T_I} \cdot \sum_{n=0}^{i} e_n$$

- precision of the digital I channel can be improved by:
  - decreasing Δt sampling time
  - using the trapezoid rule

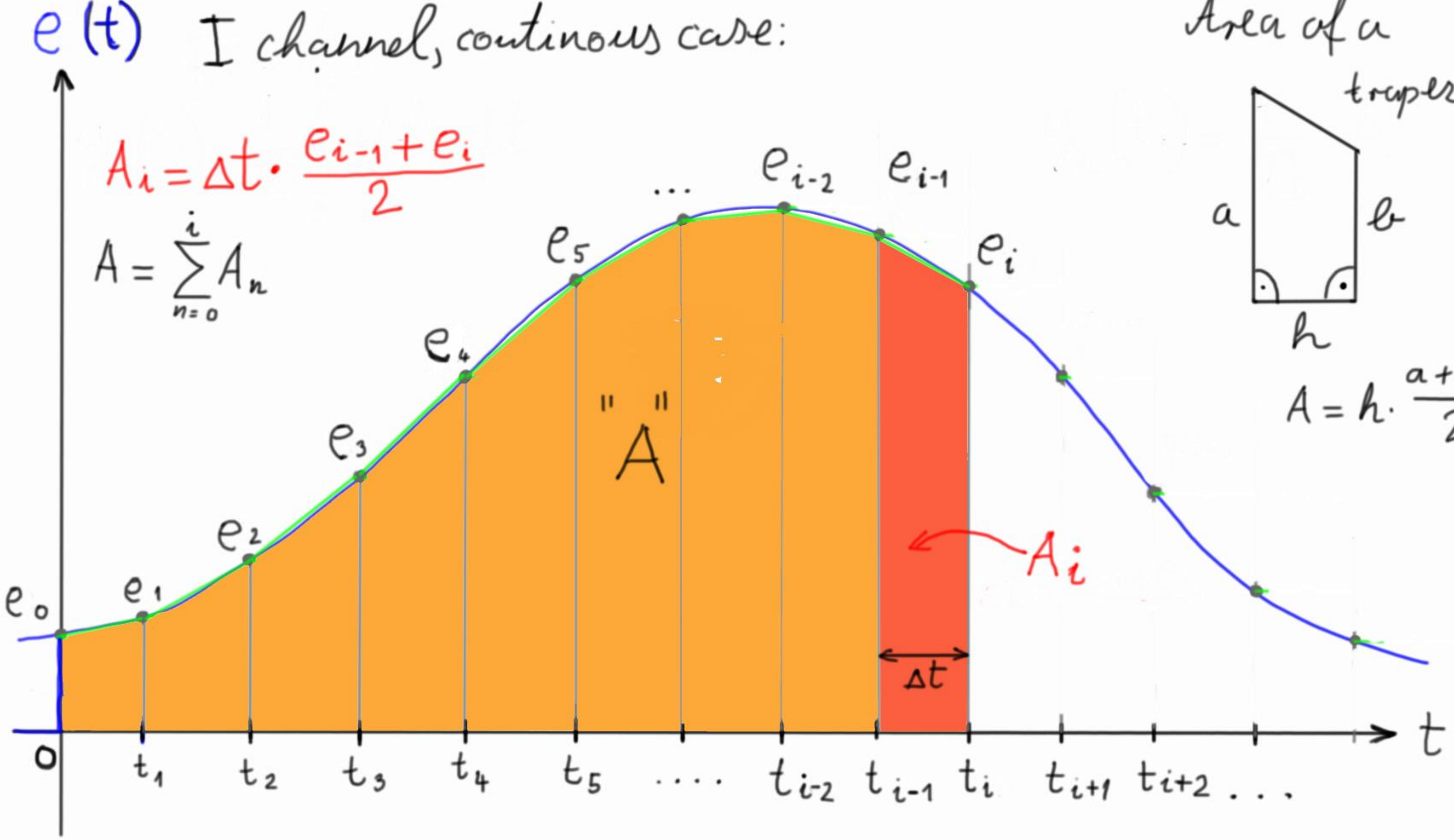


e(t)
if Δt ⟼ 0   Δt · Σ_{n=0}^{i} e_n ⟼ ∫_0^{t_i} e(t)dt
e_i
t_0
t_i
t

I channel, continous case:

$$A_i = \Delta t \cdot \frac{e_{i-1}+e_i}{2}$$

$$A = \sum_{n=0}^{i} A_n$$

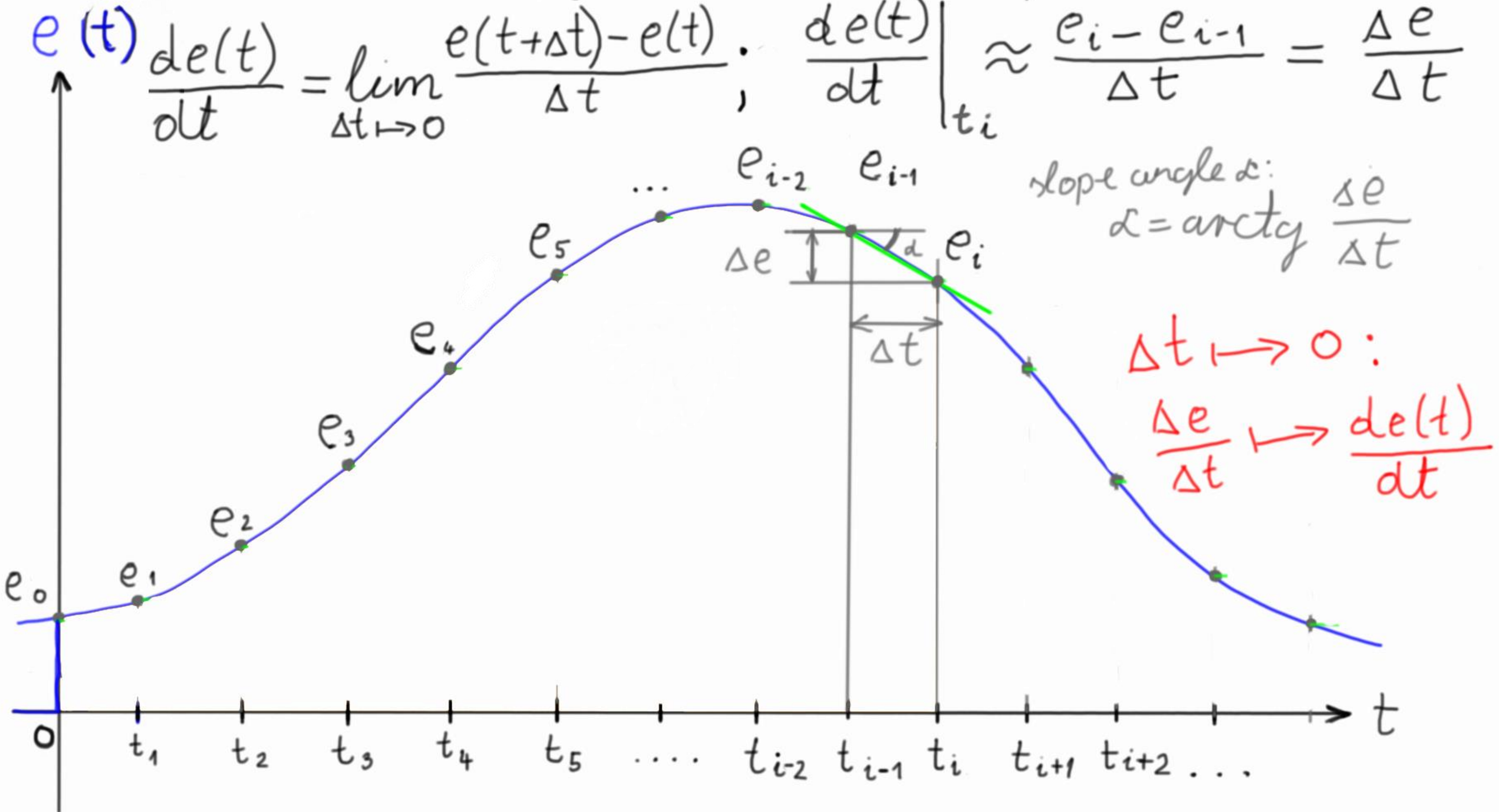


Area of a trapezoid:

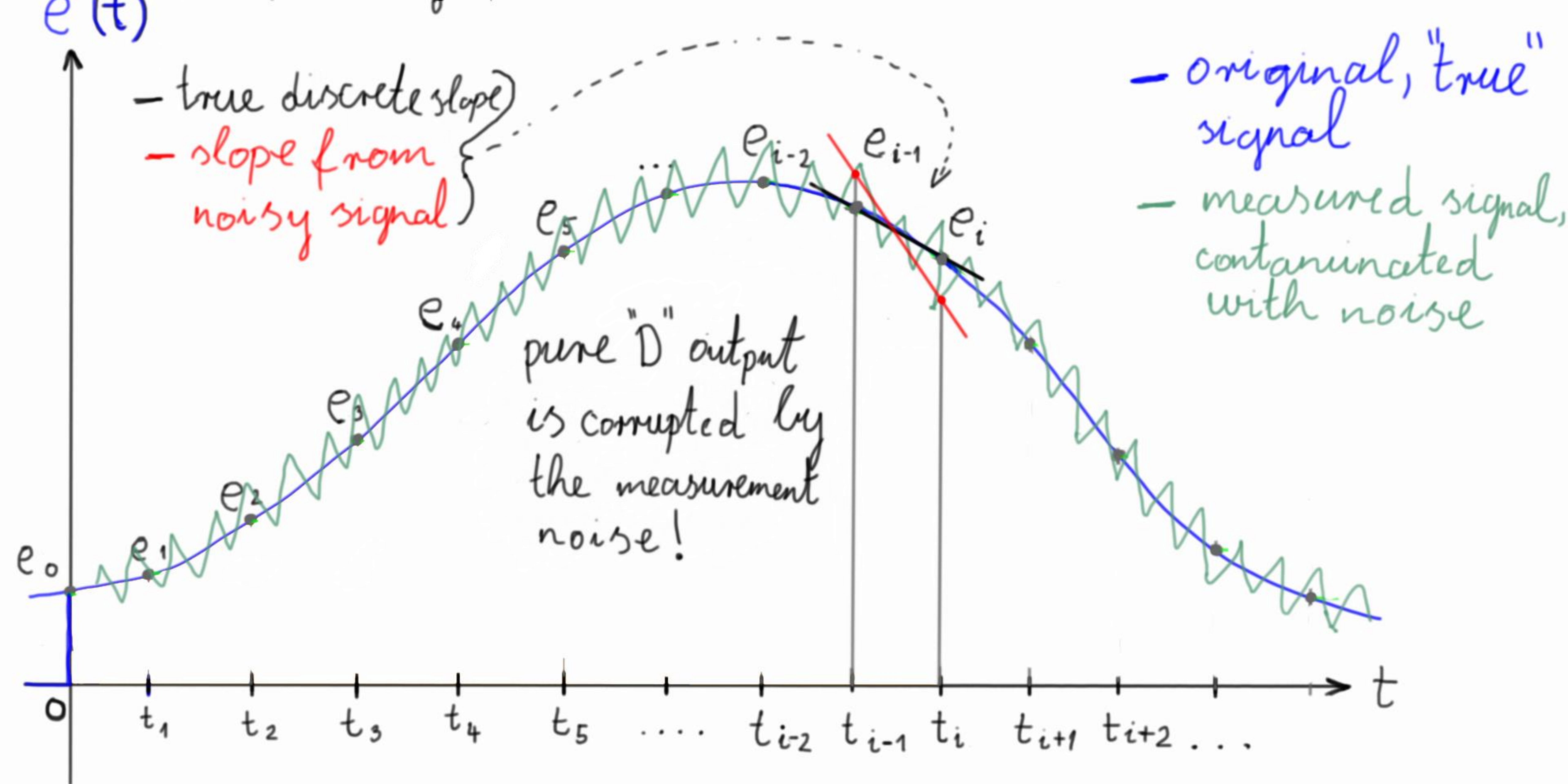
$$A = h \cdot \frac{a+b}{2}$$

Pure D channel, continous and digital case:

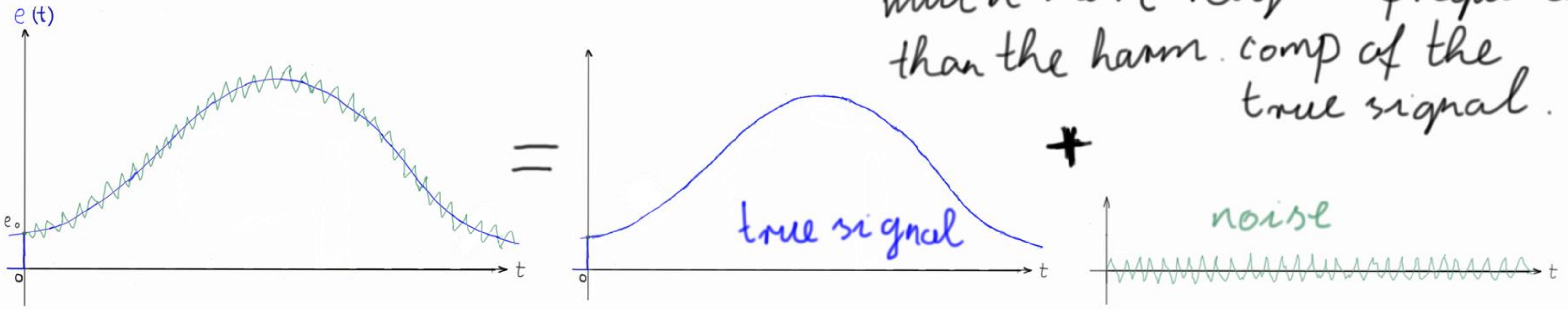
$$\frac{de(t)}{dt} = \lim_{\Delta t \mapsto 0} \frac{e(t+\Delta t) - e(t)}{\Delta t}; \quad \left.\frac{de(t)}{dt}\right|_{t_i} \approx \frac{e_i - e_{i-1}}{\Delta t} = \frac{\Delta e}{\Delta t}$$



Is it necessary to realise the T_1 part $\left(\frac{1}{sT+1}\right)$ of the D chan?



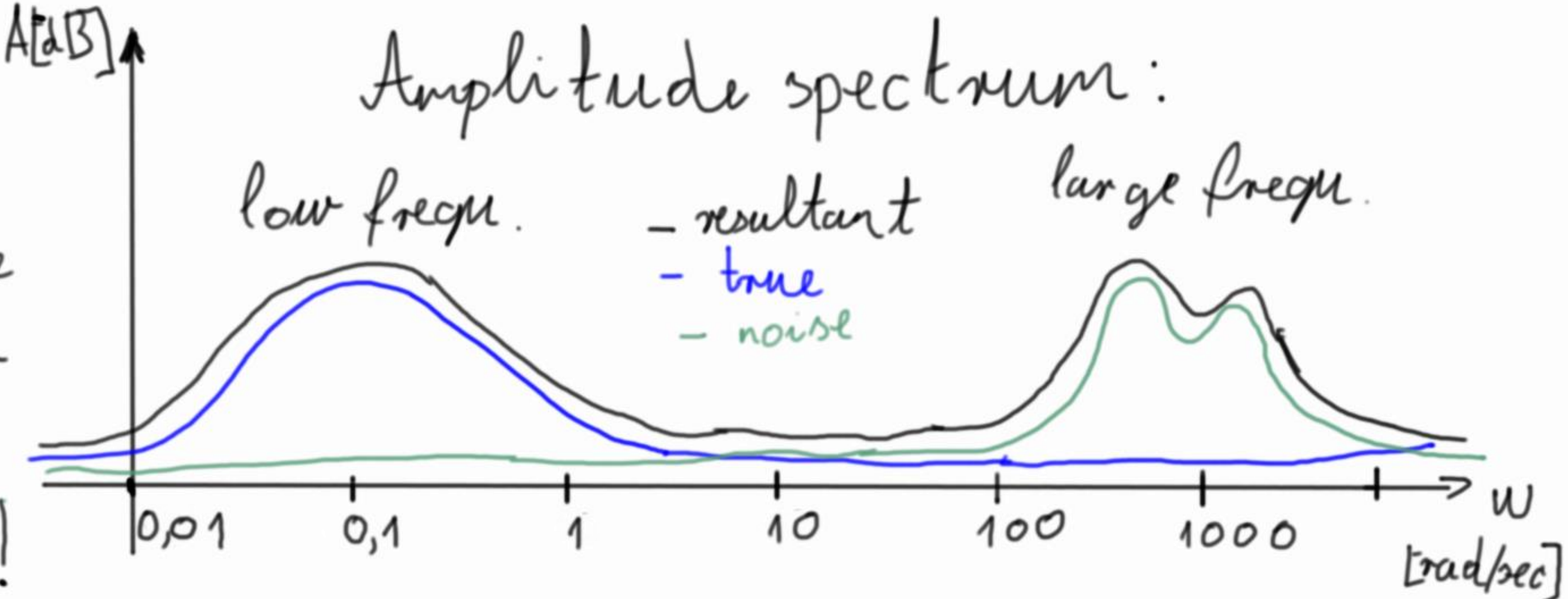
Usually, the noise is consisting of harmonic componenst wih much more higher frequencies than the harm. comp of the true signal.

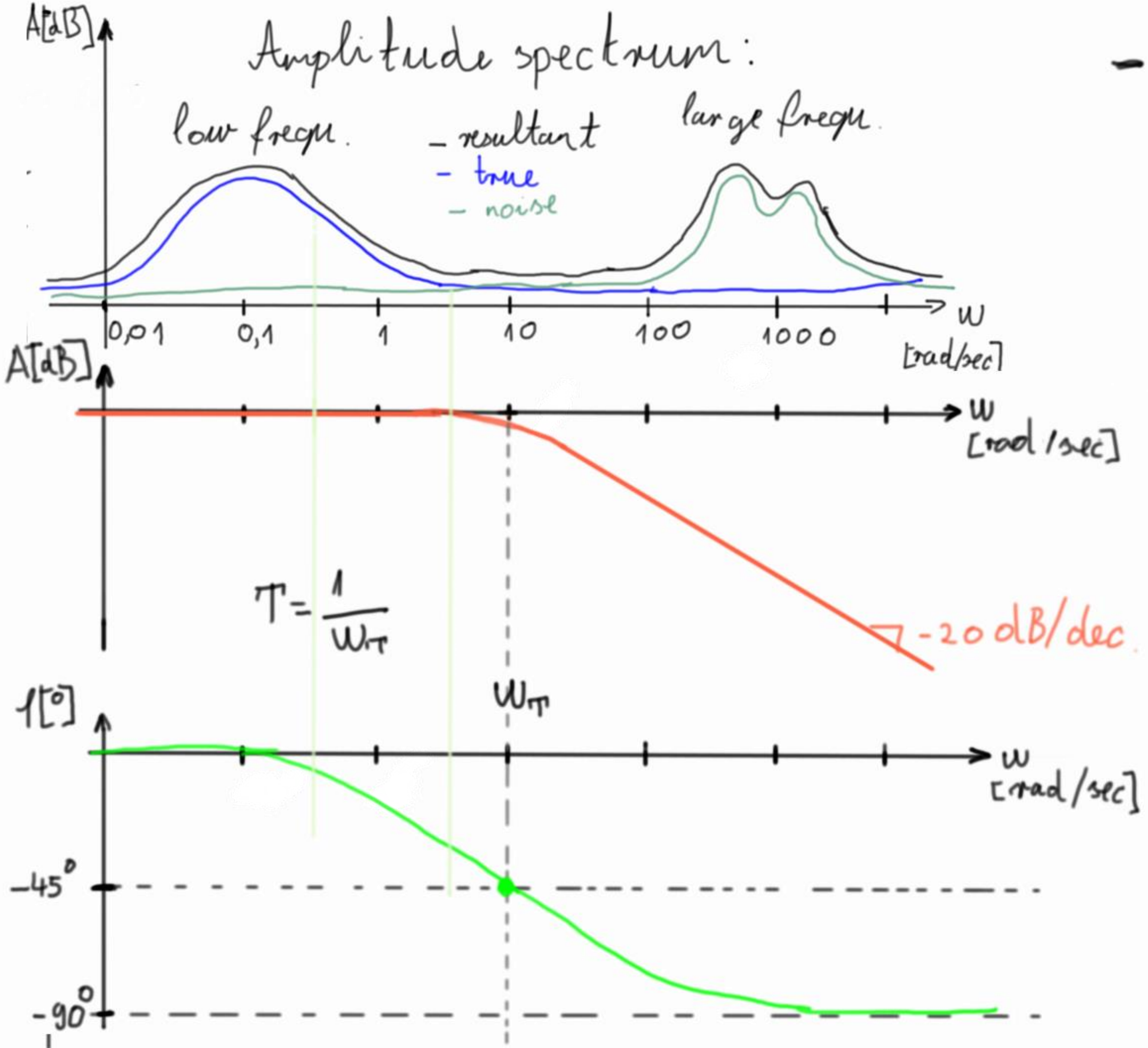


— the noise content can be eliminated by filtering out the high frequency components!

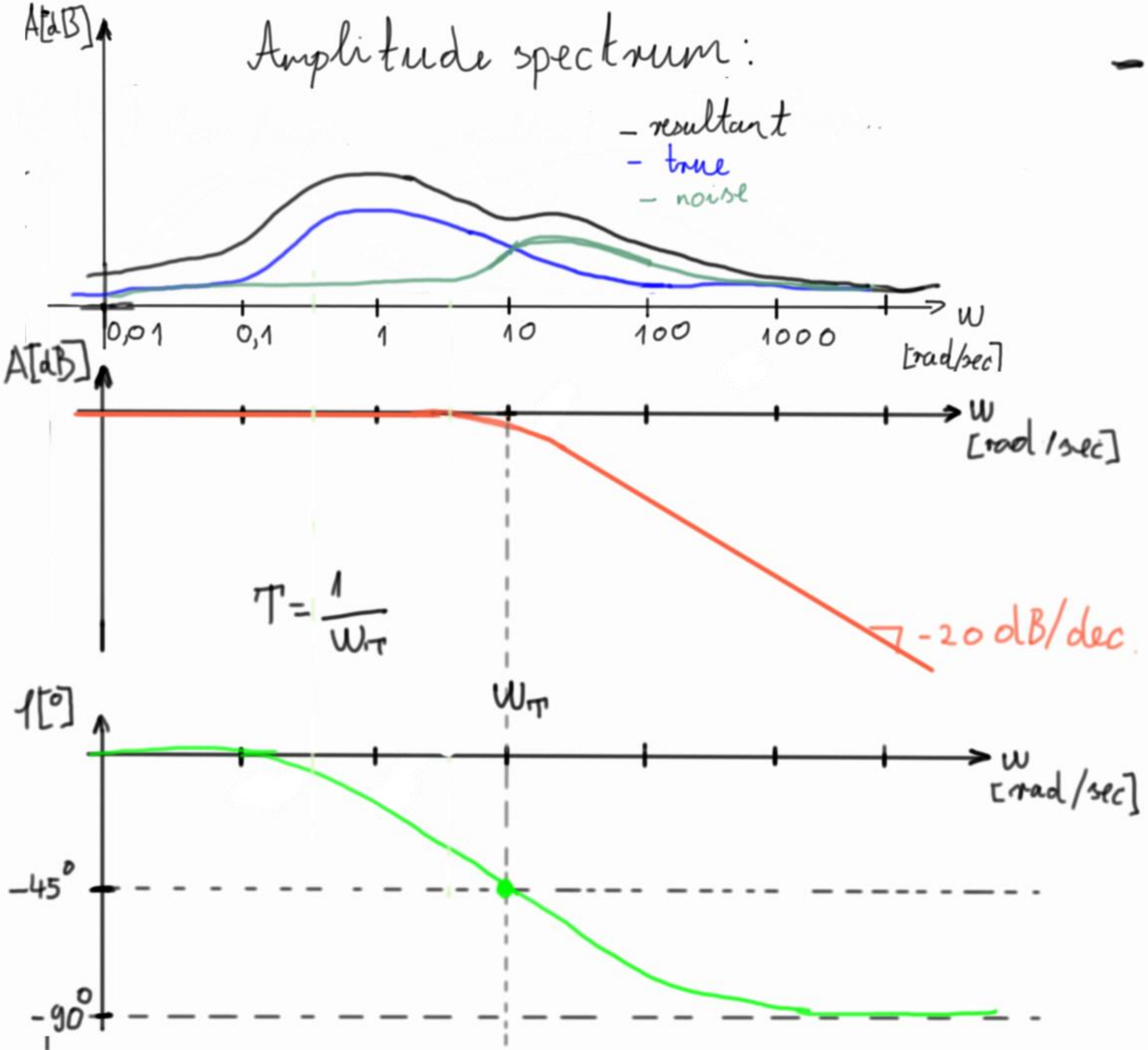
→ low pass filter!

Amplitude spectrum:





- if T (time constant of the T_1 block inside the DT_1 channel) is selected carefully, the noise content can be damped.
- however, some components of the true signal can also get some extra phashift
- if the noise and the true signal does not separates clearly, the true signal components can be atenuated, as well



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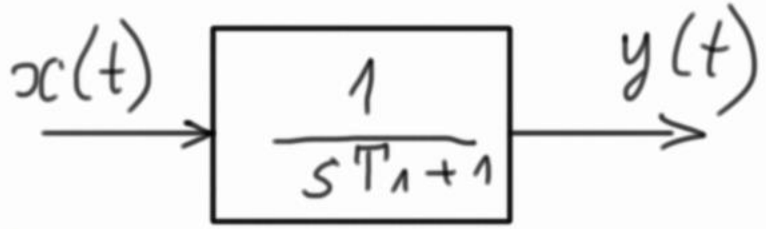
How can we realise a digital T_1 low-pass filter?

Diffe: $T\dfrac{dy(t)}{dt} + y(t) = x(t)$



$y_0(t_0=0) = 0 \mapsto$ known

$$\frac{dy(t)}{dt} \approx \frac{y_1 - y_0}{\Delta t}$$

unknown

let's use a unit step input first

$1(t) \approx 1$

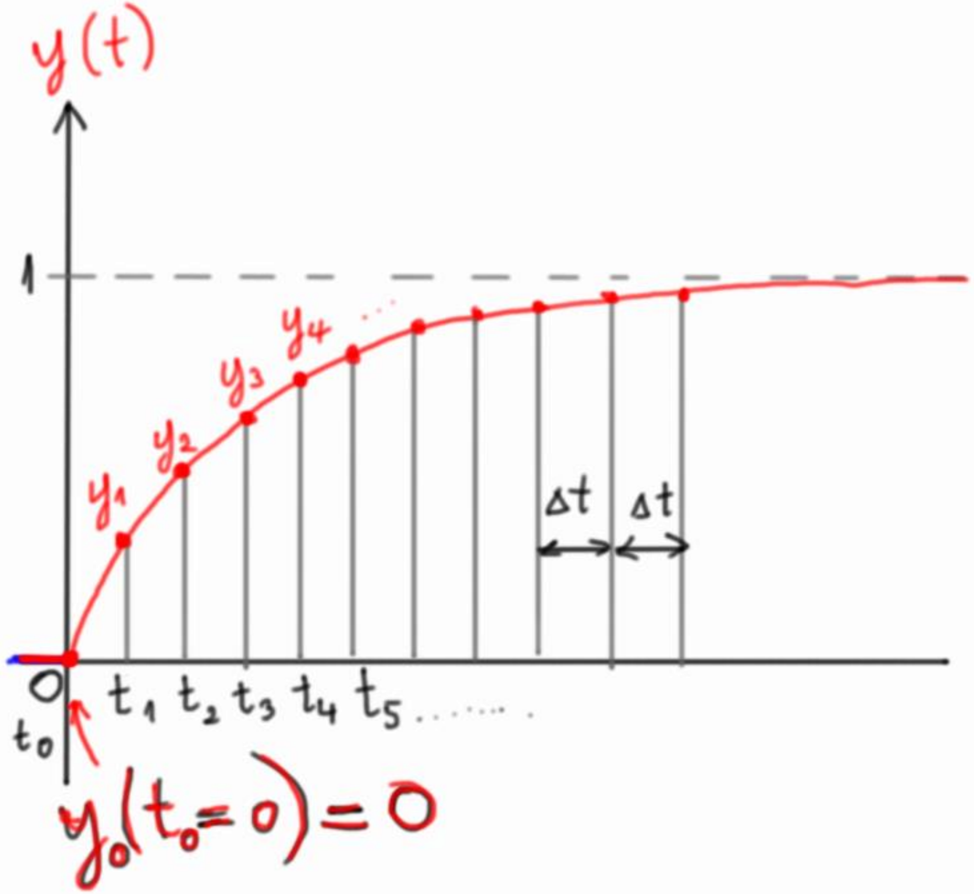
$$T \cdot \frac{y_1 - y_0}{\Delta t} + y_0 = 1(t)$$

$$T \cdot \frac{y_1 - y_0}{\Delta t} + y_0 = 1$$

$$T \cdot \frac{y_1 - y_0}{\Delta t} = 1 - y_0$$

$$(y_1 - y_0) = \frac{\Delta t}{T}(1 - y_0)$$

$$\boxed{y_1 = y_0 + \frac{\Delta t}{T}(1 - y_0)}$$



$$y_1 = y_0 + \frac{\Delta t}{T} \cdot (1 - y_0)$$

if y_1 is known, we can calc y_2:

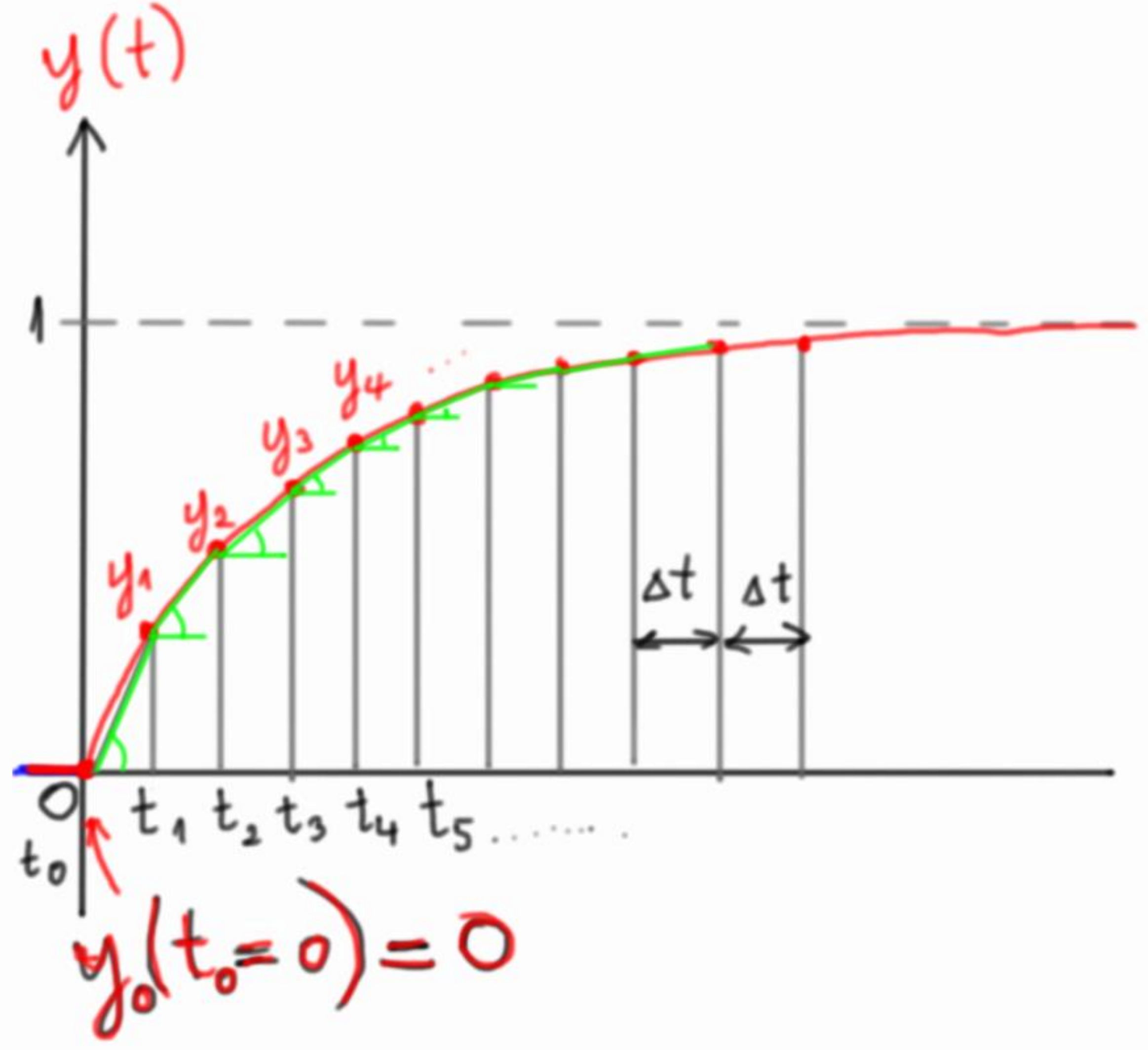
$$y_2 = y_1 + \frac{\Delta t}{T} (1 - y_1)$$

$$y_3 = y_2 + \frac{\Delta t}{T} (1 - y_2)$$

$$\underbrace{y_i}_{\text{actual}} = \underbrace{y_{i-1}}_{\text{prev}} + \underbrace{\frac{\Delta t}{T} (1 - y_{i-1})}_{\text{increase}}$$

$y \uparrow \longrightarrow \downarrow$ increase

(iterative calc method)



If the input is an arbitrary $x(t)$ function

$$y_i = y_{i-1} + \frac{\Delta t}{T}\left(x_i - y_{i-1}\right)$$

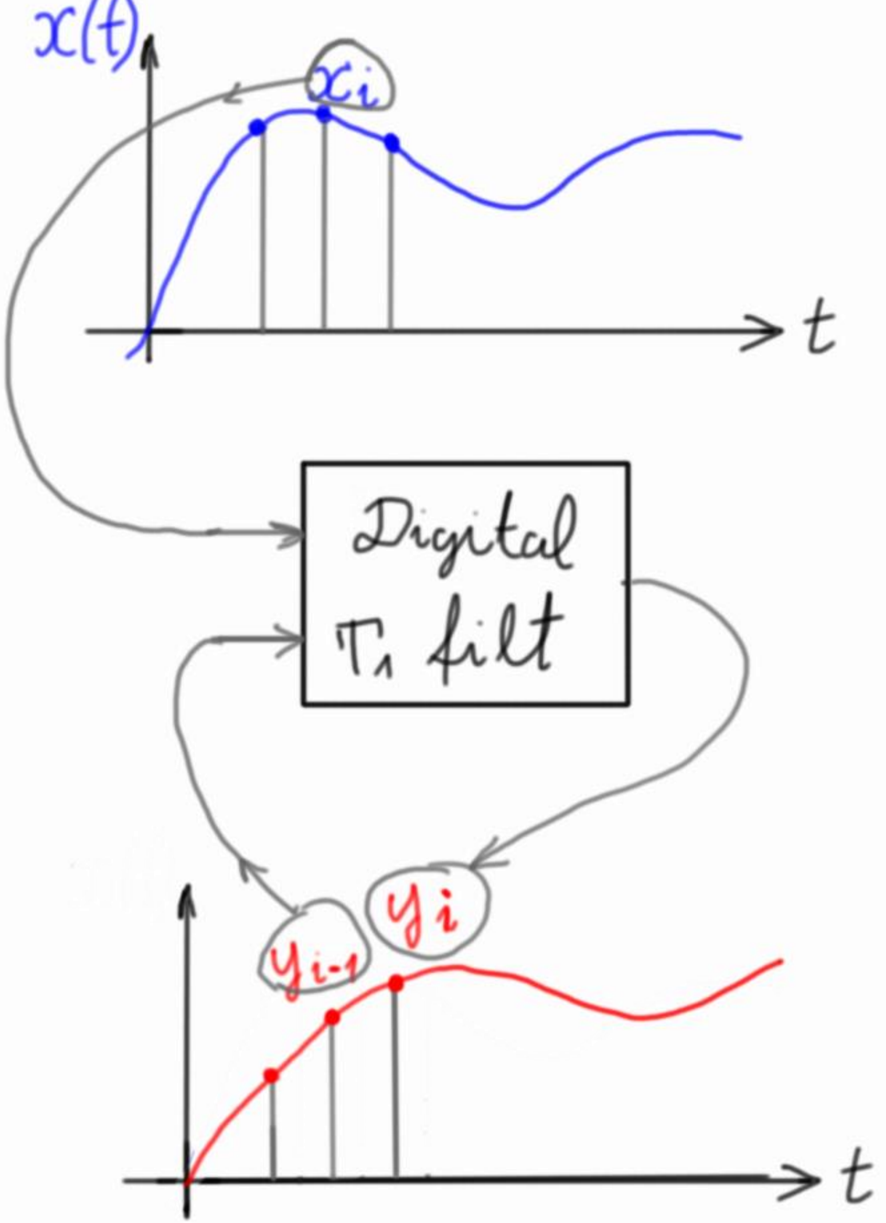
y_i: current filter output

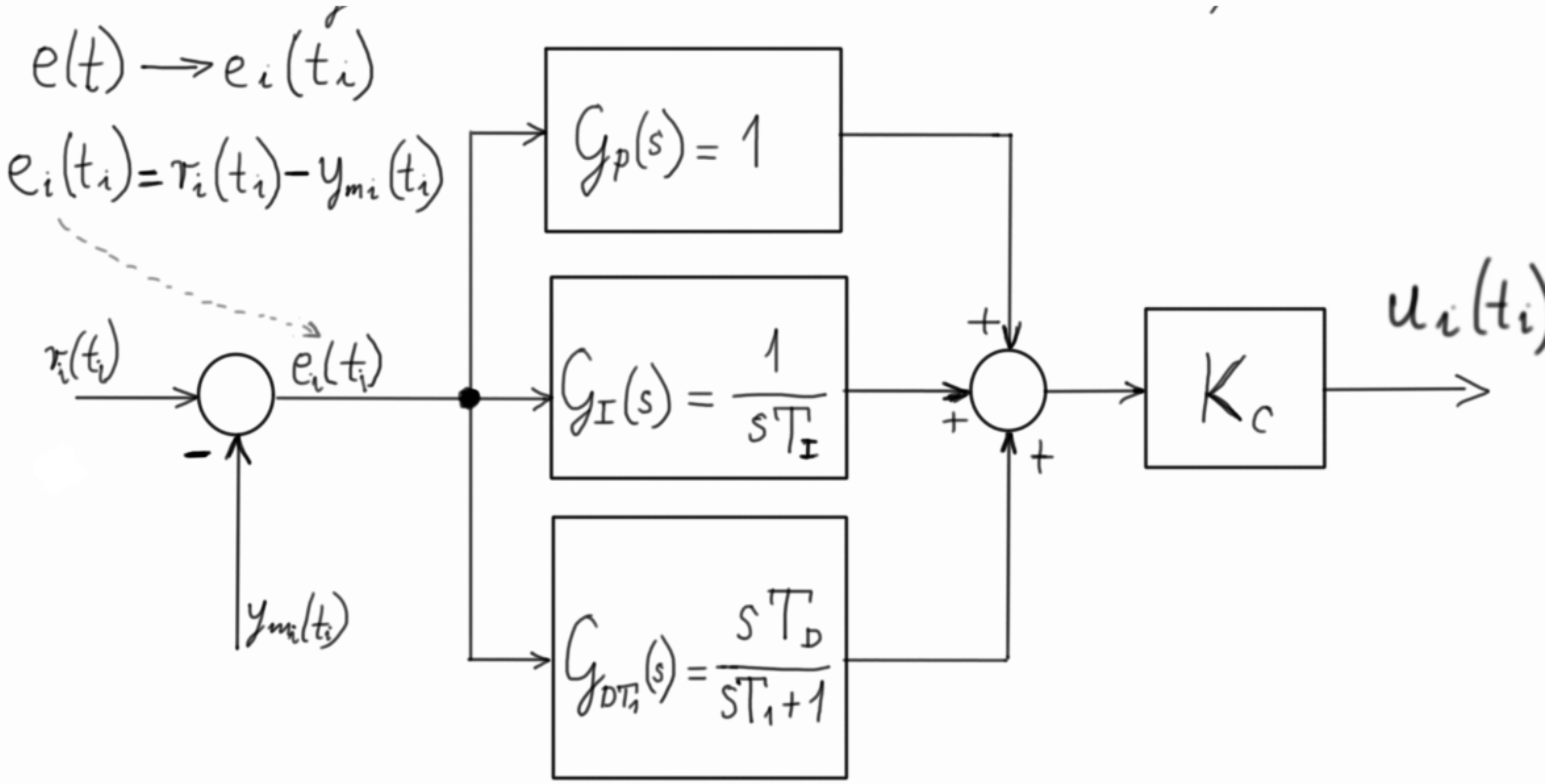
y_{i-1}: previous filter output

x_i : current filter input

DT_1 output:

$$y_{DT_1,i} = \frac{y_i - y_{i-1}}{\Delta t}$$





$$u_i(t_i) = \left[e_i(t_i) + \frac{\Delta t}{T_I} \sum_{n=0}^{i} e_i + \frac{e_{filt,i} - e_{filt\,i-1}}{\Delta t} \cdot T_D \right] \cdot K_C$$

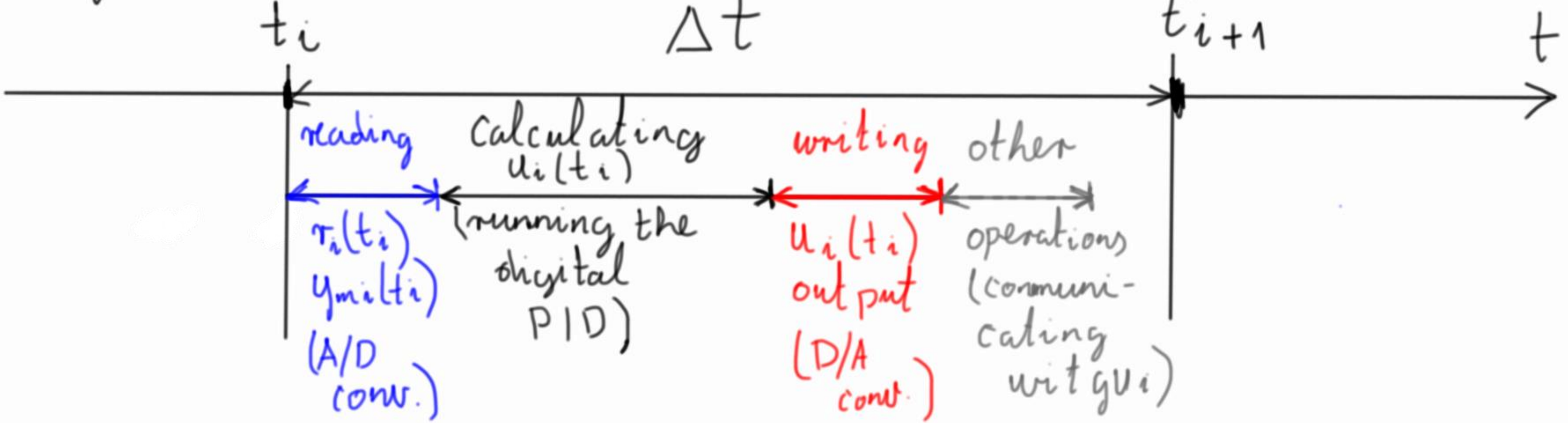
$$e_{filt,i} = e_{filt\,i-1} + \frac{\Delta t}{T} \left(e_i(t_i) - e_{filt\,i-1} \right)$$

Why not use even smaller Δt values?

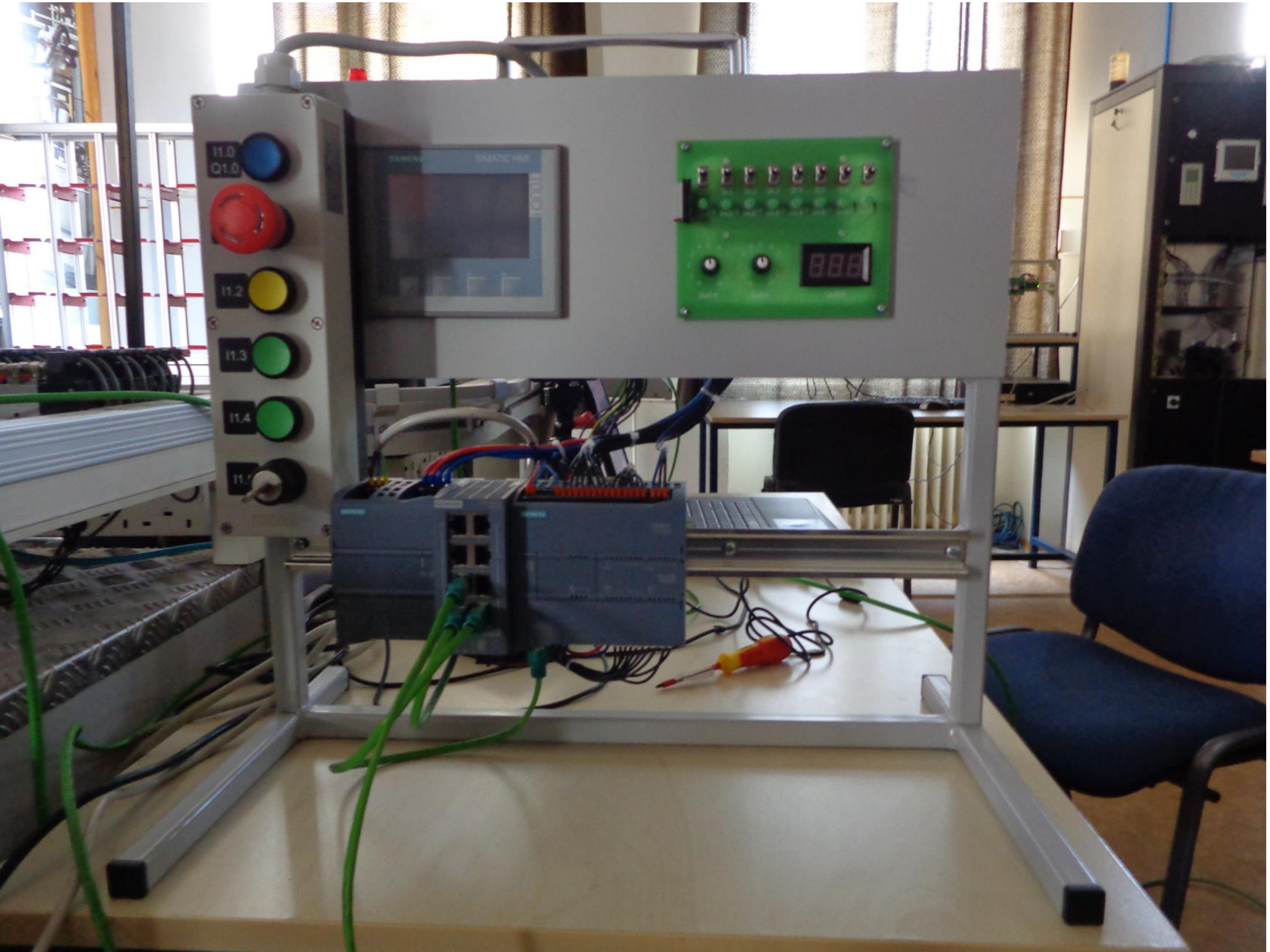
– with smaller Δt values, the D, I an T, channel outputs approximates the continous case even better...

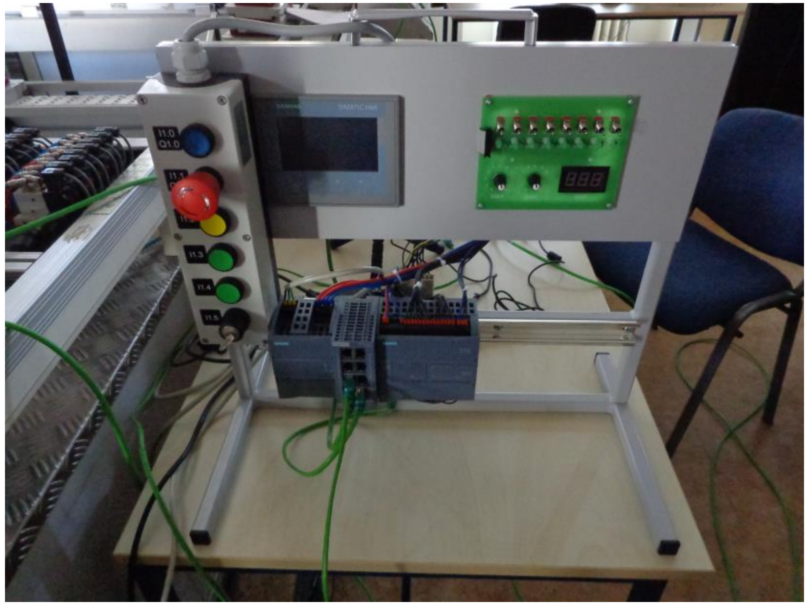
digital

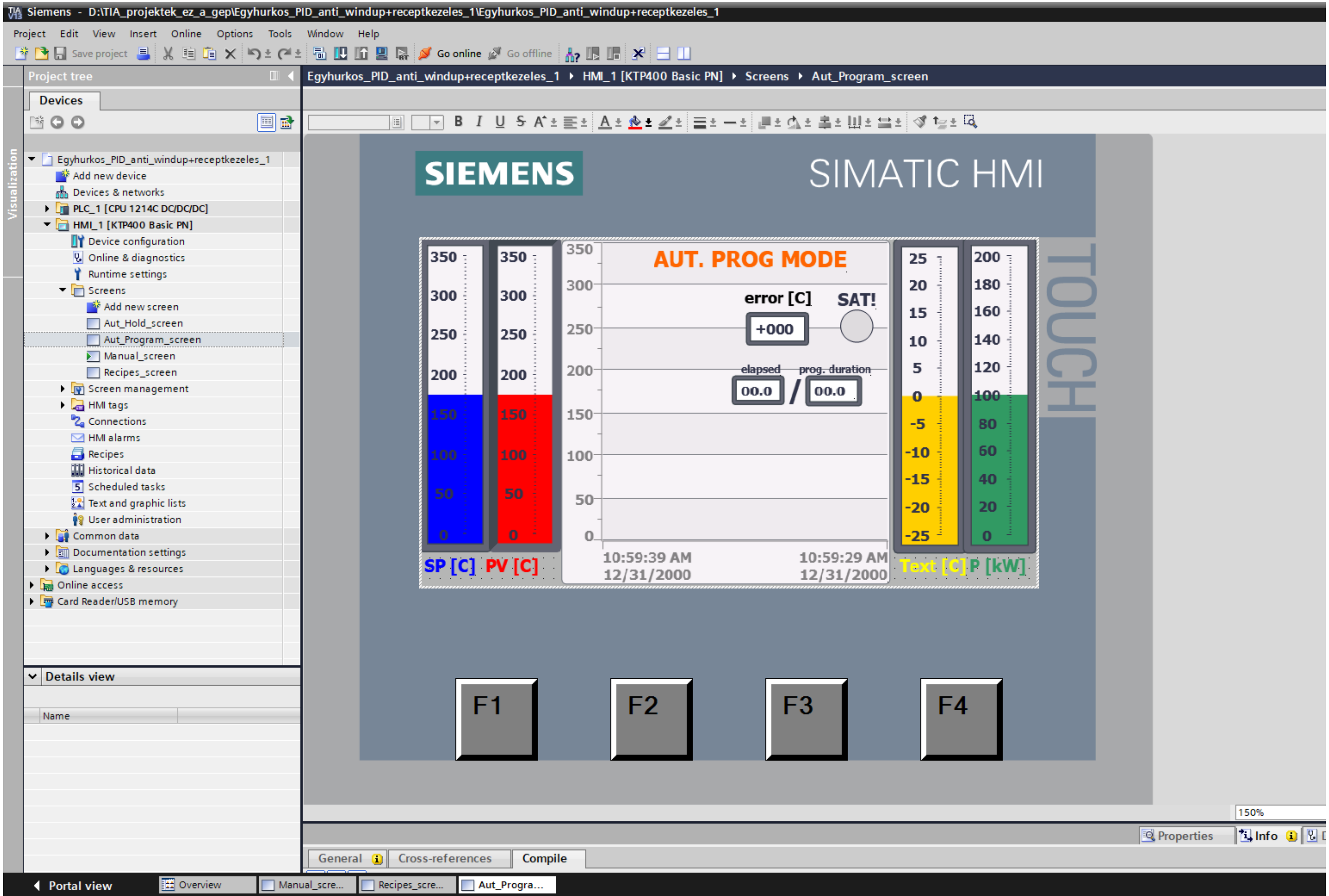
– but: the microcontroller has to finish all operations during the Δt time period!



Siemens S7 1200 PLC






Siemens - D:\TIA_projektek_ez_a_gep\Egyhurkos_PID_anti_windup+receptkezeles_1\Egyhurkos_PID_anti_windup+receptkezeles_1
Project Edit View Insert Online Options Tools Window Help
Save project
Go online
Go offline
Project tree
Devices
Egyhurkos_PID_anti_windup+receptkezeles_1
Add new device
Devices & networks
PLC_1 [CPU 1214C DC/DC/DC]
HMI_1 [KTP400 Basic PN]
Device configuration
Online & diagnostics
Runtime settings
Screens
Add new screen
Aut_Hold_screen
Aut_Program_screen
Manual_screen
Recipes_screen
Screen management
HMI tags
Connections
HMI alarms
Recipes
Historical data
Scheduled tasks
Text and graphic lists
User administration
Common data
Documentation settings
Languages & resources
Online access
Card Reader/USB memory
Details view
Name
Visualization
Egyhurkos_PID_anti_windup+receptkezeles_1 ▸ HMI_1 [KTP400 Basic PN] ▸ Screens ▸ Aut_Program_screen
SIEMENS
SIMATIC HMI
TOUCH
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10:59:39 AM
12/31/2000
10:59:29 AM
12/31/2000
SP [C]
PV [C]
Text [C]
P [kW]
25
20
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F1
F2
F3
F4
150%
Properties
Info
General
Cross-references
Compile
Portal view
Overview
Manual_scre...
Recipes_scre...
Aut_Progra...

All lab materials available at:

https://rosz.uni-obuda.hu/esemenyek/plc-s-szabalyzasok-tanfolyam/

Or:

https://classroom.google.com/c/NjIyMzYxODg4MTY5?cjc=dhc4ud7